



Bauhaus-Universität Weimar

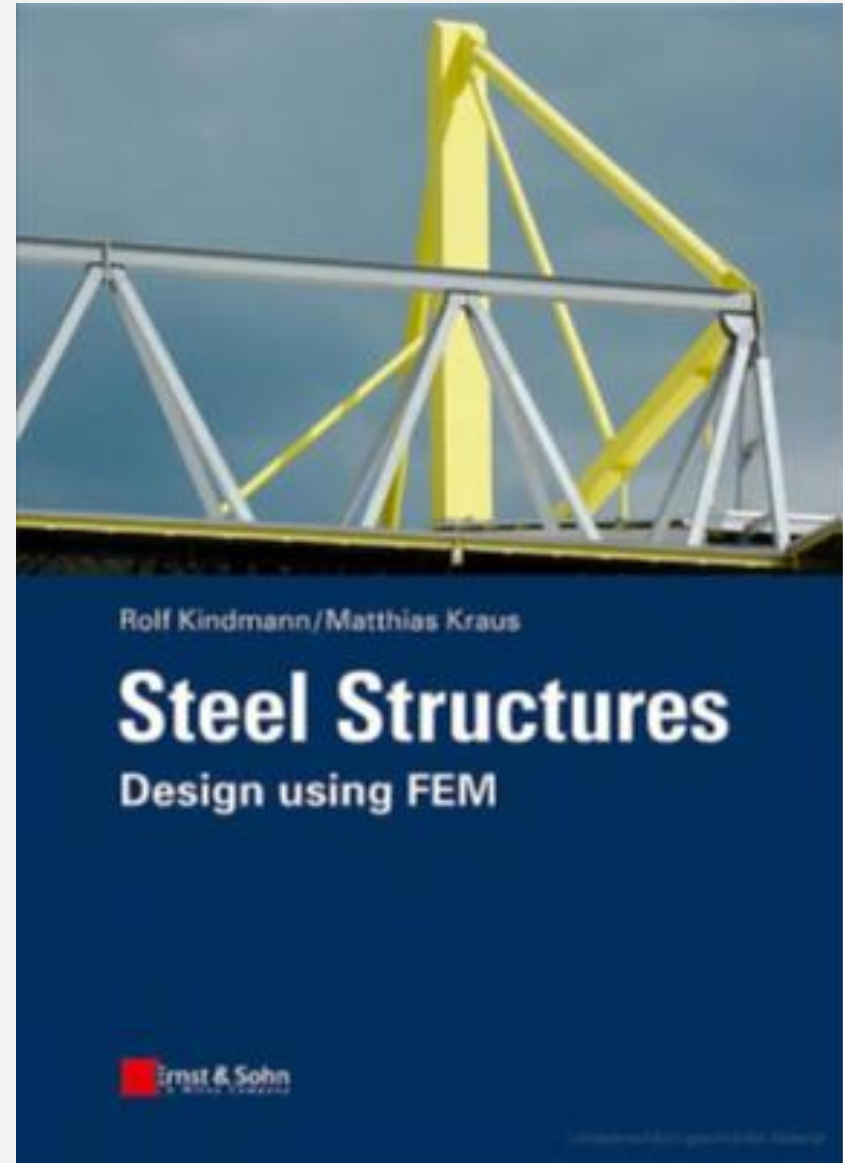
Professur Stahl- und Hybridbau

Univ.-Prof. Dr.-Ing. Matthias Kraus
Bauhaus-Universität Weimar, Chair of Steel and Hybrid Structures

1 Exercises

Module: **Numerical methods for lateral torsional buckling**
Ph.D. Course: Computational structural design (June 25th to 29th, 2018)
Presented by: Stalin Ibáñez

Kindmann, Kraus:
Steel Structures – Design using FEM



Eurocode 3: Design of steel structures

EN 1993-1: General rules and rules for buildings

EN 1993-2: Steel Bridges

EN 1993-3: Towers, masts and chimneys

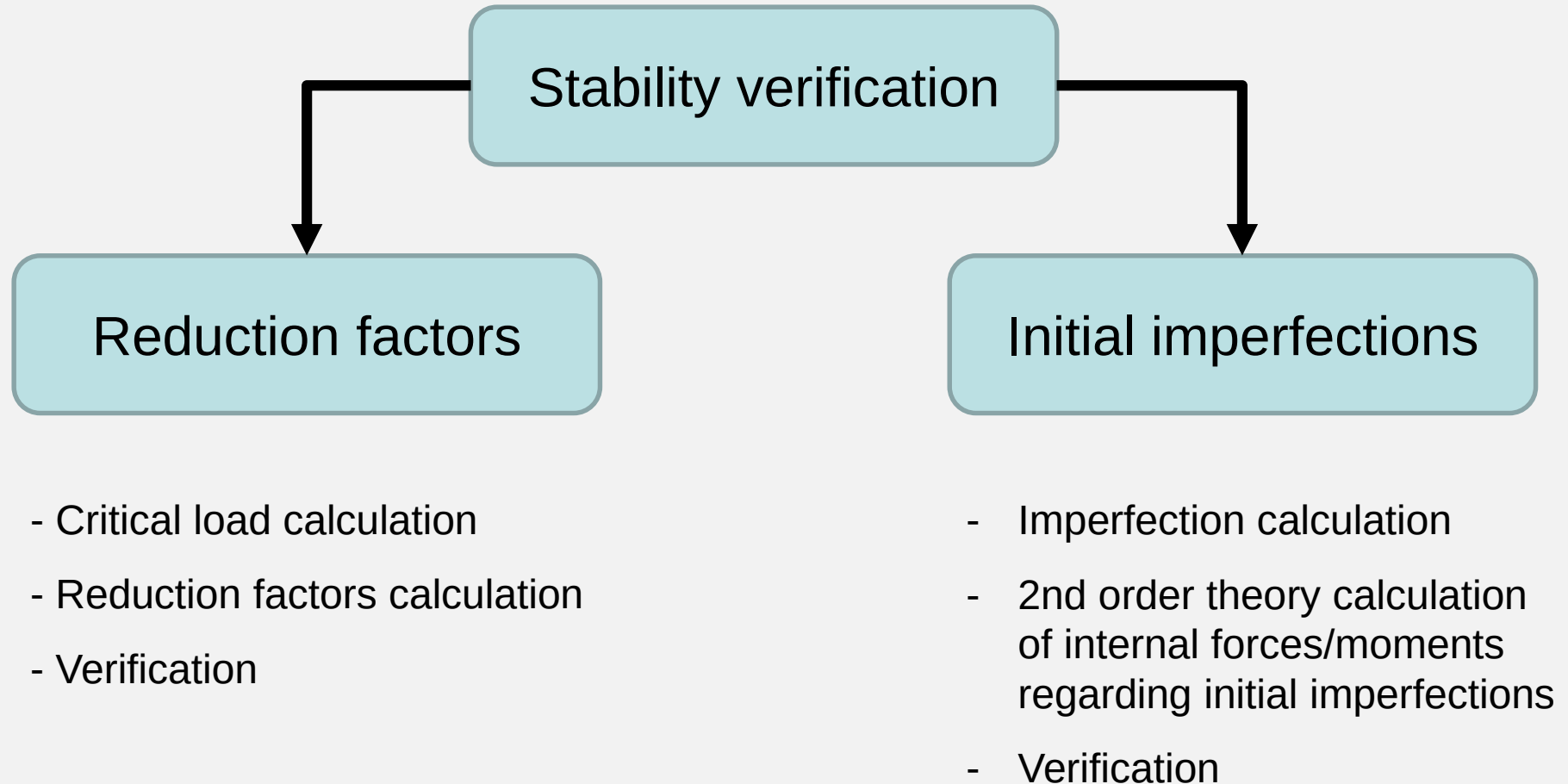
EN 1993-4: Silos and Tanks

EN 1993-5: Piling

EN 1993-6: Crane supporting structures

- EN 1993-1-1: General rules and rules for buildings
- EN 1993-1-2: Structural fire design
- EN 1993-1-3: Supplementary rules for cold-formed members and sheeting
- EN 1993-1-4: Supplementary rules for stainless steels
- EN 1993-1-5: Plated structural elements
- EN 1993-1-6: Strength and stability of shell structures;
- EN 1993-1-7: Plated structures subject to out of plane loading
- EN 1993-1-8: Design of joints
- EN 1993-1-9: Fatigue
- EN 1993-1-10: Material toughness and through-thickness properties
- EN 1993-1-11: Design of structures with tension components
- EN 1993-1-12: Additional rules for the extension of EN 1993 up to steel grades S700

Overview



Overview

1. Software presentation for structural analysis
 - 7 DOF
 - 2nd Order analysis (eigenvalues and mode shapes)
 - Plastic verification (Partial Internal Force method [1])
2. Reduction factors
 - Critical load using software
 - Hand verification
3. Initial imperfections
 - Eurocode v/s German Annex

1. Software presentation for structural analysis

- 7 DOF



Lehr- und Lernprogramme für Studium und Weiterbildung
Bauhaus-Universität Weimar - Lehrstuhl für Stahl- und Hybridbau
Prof. Dr.-Ing. M. Kraus

Program KSTAB written by R. Kindmann and J. Laumann
Ruhr-Universität Bochum

Program
version
06/2007

Deformations, Internal forces, Load-carrying capacity (TSV), Eigenvalues and Egenforms

Project: Modelling of structures
Comment: single-span girder with single load

Calculation according to 2nd Order Theory

Elasticity modulus: 21,000 η_{kl} calculation? yes
Shear modulus: 8,100 kN/cm² max. iterations 40
 $f_{y,k} =$ 35.50 kN/cm² Accuracy limit for $h_{ki} =$ 1,0E-04
 $\gamma_M =$ 1.10 für $f_{y,k}$, E, G $\eta_{kl} =$ **2.5918**

Sections

System length: 1200 cm $\Sigma v =$ 40

utilisation
of cross-
section: **0.855**

Section:	1	2	3	4	5	6	7	8	9	10
Length [cm]:	600	600								
Elements:	20	20								

1. Software presentation for structural analysis

- 7 DOF

	u_{Sa}	v_{Ma}	φ_{za}	w_{Ma}	φ_{ya}	ϑ_a	$-\vartheta'_a$	u_{Sb}	v_{Mb}	φ_{zb}	w_{Mb}	φ_{yb}	ϑ_b	$-\vartheta'_b$
$\delta u_{Sa} :$	EA							EA						
$\delta v_{Ma} :$		EI_z, c_v, S				c_v, S			EI_z, c_v, S				c_v, S	
$\delta \varphi_{za} :$														
$\delta w_{Ma} :$				EI_y, c_w							EI_y, c_w			
$\delta \varphi_{ya} :$														
$\delta \vartheta_a :$						$EI_\omega, GI_T, c_\vartheta,$							$EI_\omega, GI_T, c_\vartheta,$	
$-\delta \vartheta'_a :$						c_v, S			c_v, S				c_v, S	
$\delta u_{Sb} :$								EA						
$\delta v_{Mb} :$									EI_z, c_v, S				c_v, S	
$\delta \varphi_{zb} :$														
$\delta w_{Mb} :$		sym.									EI_y, c_w			
$\delta \varphi_{yb} :$														
$\delta \vartheta_b :$													$EI_\omega, GI_T, c_\vartheta,$	
$-\delta \vartheta'_b :$													c_v, S	

1. Software presentation for structural analysis

- 7 DOF

Supports and single springs			-1=fixed; 0=free; value>0: spring; -2 in column 1= all fixed (in the bar)							
Position x =	0	600	1,200							
$u_s =$	-1	0	0							
$v_M =$	-1	-1	-1							
$v_M' =$	0	0	0							
$w_M =$	-1	-1	-1							
$w_M' =$	0	0	0							
$\vartheta =$	-1	-1	-1							
$\vartheta' =$	0	0	0							
Standard supp.										

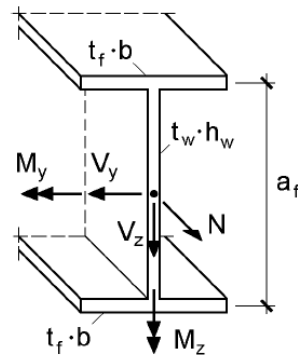
1. Software presentation for structural analysis

- 2nd Order analysis (eigenvalues and mode shapes)

	u_{Sa}	v_{Ma}	φ_{za}	w_{Ma}	φ_{ya}	ϑ_a	$-\vartheta'_a$	u_{Sb}	v_{Mb}	φ_{zb}	w_{Mb}	φ_{yb}	ϑ_b	$-\vartheta'_b$
$\delta u_{Sa} :$														
$\delta v_{Ma} :$		N				$M_y,$ $z_M \cdot N$			N				$M_y,$ $z_M \cdot N$	
$\delta \varphi_{za} :$														
$\delta w_{Ma} :$				N		$M_z,$ $y_M \cdot N$					N		$M_z,$ $y_M \cdot N$	
$\delta \varphi_{ya} :$														
$\delta \vartheta_a :$						$M_{rr}, q_y \cdot y_q,$			$M_y,$		$M_z,$		$M_{rr}, q_y \cdot y_q,$	
$-\delta \vartheta'_a :$						$q_z \cdot z_q$			$z_M \cdot N$		$y_M \cdot N$		$q_z \cdot z_q$	
$\delta u_{Sb} :$														
$\delta v_{Mb} :$									N				$M_y,$ $z_M \cdot N$	
$\delta \varphi_{zb} :$														
$\delta w_{Mb} :$											N		$M_z,$ $y_M \cdot N$	
$\delta \varphi_{yb} :$														
$\delta \vartheta_b :$													$M_{rr}, q_y \cdot y_q,$	
$-\delta \vartheta'_b :$													$q_z \cdot z_q$	

1. Software presentation for structural analysis

- Plastic verification



Verification conditions (partial internal forces method):

Shear force V_y : $V_y \leq V_{pl,y} = 2 \cdot b \cdot t_f \cdot f_y / \sqrt{3}$

Shear force V_z : $V_z \leq V_{pl,z} = h_w \cdot t_w \cdot f_y / \sqrt{3}$

Bending moment M_z :

$$M_z \leq M_{pl,z,\tau} = f_y \cdot t_f \cdot b^2 / 2 \cdot \sqrt{1 - (V_y / V_{pl,y})^2}$$

Axial force N and bending moment M_y :

$$N < N_{gr,w} : M_y \leq N_{gr,f} \cdot a_f + M_w$$

or

$$N_{gr,w} \leq N \leq N_{gr} : M_y \leq (N_{gr} - N) \cdot a_f / 2$$

Insert all internal forces **according to magnitude!**

Calculation values:

$$N_{gr,f} = t_f \cdot b \cdot f_y \cdot \sqrt{1 - (V_y / V_{pl,y})^2} \cdot \sqrt{1 - M_z / M_{pl,z,\tau}}$$

$$N_{gr} = 2 \cdot N_{gr,f} + N_{gr,w}$$

$$N_{gr,w} = t_w \cdot h_w \cdot f_y \cdot \sqrt{1 - (V_z / V_{pl,z})^2}$$

$$M_w = \left(1 - (N / N_{gr,w})^2\right) \cdot N_{gr,w} \cdot h_w / 4$$

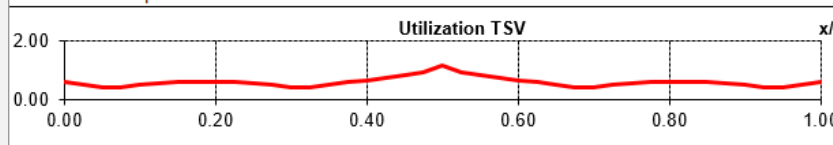
Cross-sectional load-carrying capacity acc. to the partial internal forces method TSV

Number of failed points:

2.00

max. utilization:

1.164



2. Reduction factors

Procedure:

1. Calculation of critical load

$$N_{cr} = ?$$

$$M_{cr} = ?$$

2. Calculation of slenderness

$$\bar{\lambda} = \sqrt{\frac{N_{pl}}{N_{cr}}}$$

$$\bar{\lambda}_{LT} = \sqrt{\frac{M_{pl}}{M_{cr}}}$$

3. Calculation of reduction factor

$$\varphi \longrightarrow \chi$$

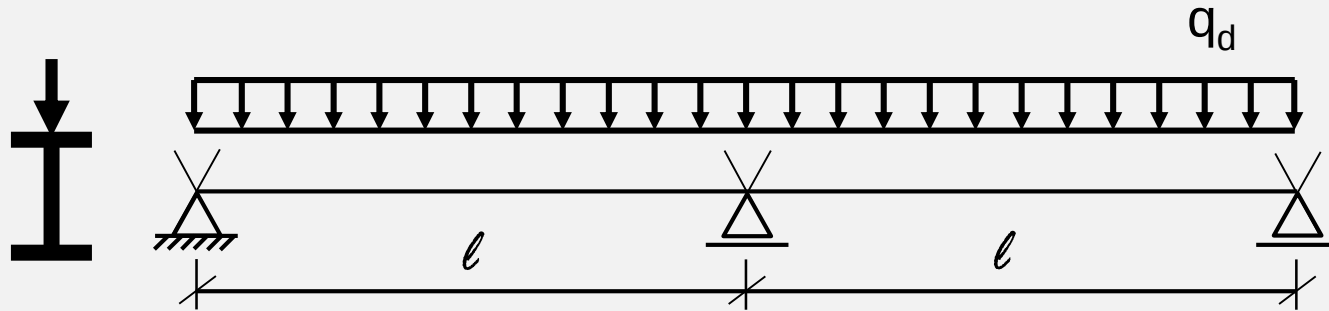
$$\varphi_{LT} \longrightarrow \chi_{LT}$$

4. Verification

$$N_{b,Rd} = \frac{\chi \cdot N_{pl}}{\gamma_{M1}}$$

$$M_{b,Rd} = \frac{\chi_{LT} \cdot M_{pl}}{\gamma_{M1}}$$

2. Reduction factors



HEB 360

q_d : 150 kN/m

L : 6 m

f_y : 35.5 kN/cm²

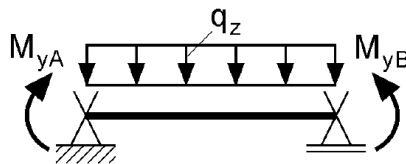
2. Reduction factors

Hand calculation

$$M_{cr,y0} = \zeta_0 \cdot N_{cr,z} \cdot \left(\zeta_0 \cdot 0.4 \cdot z_q + \sqrt{(\zeta_0 \cdot 0.4 \cdot z_q)^2 + c^2} \right)$$

$$c^2 = \frac{I_\omega + 0.039 \cdot l^2 \cdot I_T}{I_z}$$

ζ_0 :



$$M_{y0} = \frac{q_z \cdot l^2}{8}$$

$$\max |M_{cr,y}| = M_{cr,y0} \cdot \frac{\max |M_y|}{M_{y0}}$$

ψ or $1/\psi$		$M_{yA} = 0$	$M_{yA} = M_{yB}/2$	$M_{yA} = M_{yB}$	Eigenmodes
$\psi = \frac{M_{yB}}{M_{y0}}$	0	1.12	1.12	1.12	Lateral torsional buckling due to positive bending moments: $v(x)$ and $\vartheta(x)$ are single-waved functions and have equal signs.
	-0.1	1.19	1.22	1.26	
	-0.2	1.26	1.34	1.44	
	-0.3	1.34	1.49	1.67	
	-0.4	1.43	1.67	2.00	
	-0.5	1.53	1.90	2.46	
	-0.6	1.64	2.19	3.17	
	-0.7	1.76	2.57	4.30	
	-0.8	1.91	3.09	5.61	
	-0.9	2.06	3.78	5.15	
-1.0	2.24	4.43	4.10	Transition area: In the area of max ζ_0 $v(x)$ or $\vartheta(x)$ change the sign. The course of $v(x)$ is partly multi-waved.	
$\frac{1}{\psi} = \frac{M_{y0}}{M_{yB}}$	-0.9	2.42	4.19		3.12
	-0.8	2.66	3.42		2.31
	-0.7	2.78	2.63		1.68
	-0.6	2.38	1.93		1.21
	-0.5	1.80	1.35		0.87
	-0.4	1.26	0.91		0.60
	-0.3	0.82	0.58		0.40
	-0.2	0.47	0.33		0.24
	-0.1	0.20	0.14		0.11
	$M_{y0} \rightarrow 0$: ζ according to Table 9.15, system 4				

2. Reduction factors

Hand calculation

ζ_0 :

$$M_{yA} = 0$$

$$M_{yB} = -675 \text{ kN} \cdot \text{m}$$

$$M_{y0} = \frac{150 \cdot 6^2}{8} = 675 \text{ kN} \cdot \text{m}$$

$$\zeta_0 = 2.24$$

ψ or $1/\psi$		$M_{yA} = 0$	$M_{yA} = M_{yB}/2$	$M_{yA} = M_{yB}$	Eigenmodes
$\psi = \frac{M_{yB}}{M_{y0}}$	0	1.12	1.12	1.12	Lateral torsional buckling due to positive bending moments: $v(x)$ and $\vartheta(x)$ are single-waved functions and have equal signs.
	-0.1	1.19	1.22	1.26	
	-0.2	1.26	1.34	1.44	
	-0.3	1.34	1.49	1.67	
	-0.4	1.43	1.67	2.00	
	-0.5	1.53	1.90	2.46	
	-0.6	1.64	2.19	3.17	
	-0.7	1.76	2.57	4.30	
	-0.8	1.91	3.09	5.61	
	-0.9	2.06	3.78	5.15	
	-1.0	2.24	4.43	4.10	Transition area: In the area of max ζ_0 $v(x)$ or $\vartheta(x)$ change the sign. The course of $v(x)$ is partly multi-waved.
$\frac{1}{\psi} = \frac{M_{y0}}{M_{yB}}$	-0.9	2.42	4.19	3.12	
	-0.8	2.66	3.42	2.31	
	-0.7	2.78	2.63	1.68	
	-0.6	2.38	1.93	1.21	
	-0.5	1.80	1.35	0.87	
	-0.4	1.26	0.91	0.60	
	-0.3	0.82	0.58	0.40	
	-0.2	0.47	0.33	0.24	
	-0.1	0.20	0.14	0.11	

$M_{y0} \rightarrow 0$: ζ according to Table 9.15, system 4

$$c^2 = \frac{2\,829\,300 + 0.039 \cdot 600^2 \cdot 297.9}{10\,141} = 691.43 \text{ cm}^2$$

$$M_{cr,y0} = 2.24 \cdot 5838 \cdot \left(2.24 \cdot 0.4 \cdot (-18) + \sqrt{(2.24 \cdot 0.4 \cdot (-18))^2 + 691.43^2} \right) = 1924.98 \text{ kN} \cdot \text{m}$$

$$\max|M_{cr,y}| = 1924.98 \cdot \frac{\max|-675|}{675} = 1924.98 \text{ kN} \cdot \text{m}$$

2. Reduction factors

Hand calculation

- Slenderness

$$\lambda_{LT} = \sqrt{\frac{952.4}{1924.98}} = 0.703$$

- Reduction factors

$$\chi_{LT} = 0.868 \quad K_c = 0.91 \quad f = 0.956$$

$$\chi_{LT,mod} = \frac{0.868}{0.956} = 0.908$$

- Verification

$$M_{b,RD} = \frac{952.4 \cdot 0.908}{1.1} = 786.16 \text{ kN} \cdot \text{m}$$

$$\frac{M_d}{M_{b,RD}} = \frac{675}{786.16} = 0.86 < 1.00$$

2. Reduction factors

Software calculation

$$\max|M_{cr,y}| = \eta_{cr} \cdot \max|M_y|$$

$$\max|M_{cr,y}| = 2.852 \cdot 675 \text{ kN} \cdot \text{m}$$

$$\max|M_{cr,y}| = 1924.97 \text{ kN} \cdot \text{m}$$

Verification procedure is as in previous slide

What happens when:

The load is in a different position?

Only one span is loaded?

3. Initial imperfections

- Eurocode v/s German Annex

Table 5.1: Design values of initial local bow imperfection e_0/L (in EN1993)

Buckling curve acc. to Table 6.1	elastic analysis	plastic analysis
	e_0/L	e_0/L
a_0	1/350	1/300
a	1/300	1/250
b	1/250	1/200
c	1/200	1/150
d	1/150	1/100

(3) For a second order analysis taking account of lateral torsional buckling of a member in bending the imperfections may be adopted as $ke_{0,d}$, where $e_{0,d}$ is the equivalent initial bow imperfection of the weak axis of the profile considered. In general an additional torsional imperfection need not to be allowed for.

NOTE The National Annex may choose the value of k . **The value $k = 0.5$** is recommended.

3. Initial imperfections

- Eurocode v/s German Annex

Cross section	Dimensions	Cross section utilization	
		Elastic e_0/L	plastic e_0/L
rolled I-profiles	$h/b \leq 2.0$	1/250	1/200
	$h/b > 2.0$	1/200	1/150
welded I-profiles	$h/b \leq 2.0$	1/200	1/150
	$h/b > 2.0$	1/150	1/100
For lateral torsional buckling $v_{0,d} = e_{0,d}$ has to be applied. For $\bar{\lambda}_{LT} < 0.7$ and $\bar{\lambda}_{LT} > 1.3$ values may be divided in halves			

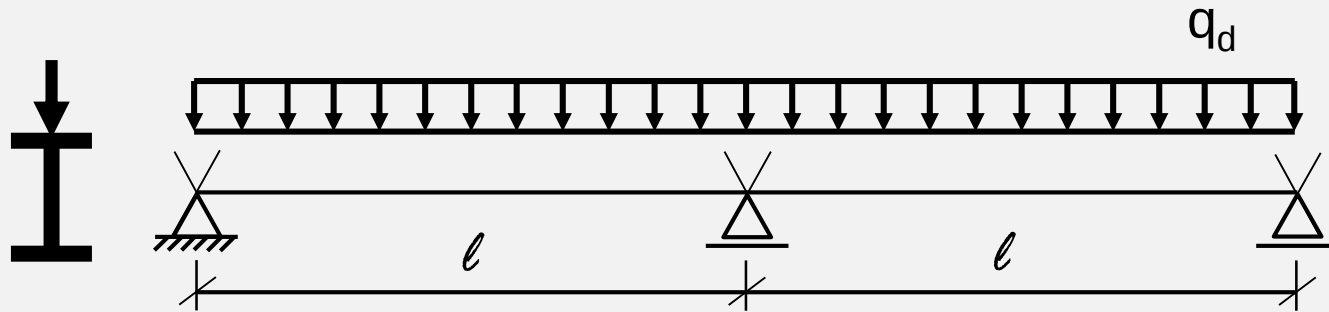
Equivalent bow imperfection for members according to DIN EN 1993-1-1 NA

3. Initial imperfections

Software calculation

- Determination of eigenmodes
- Calculation of initial imperfection
- Introduction of imperfection in the software
- 2nd order theory calculation of internal forces/moments regarding initial imperfections
- Verification using PIF method

3. Initial imperfections



HEB 360

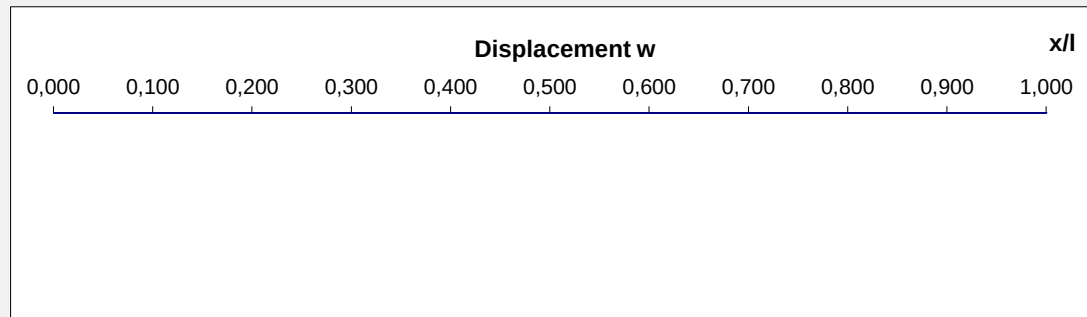
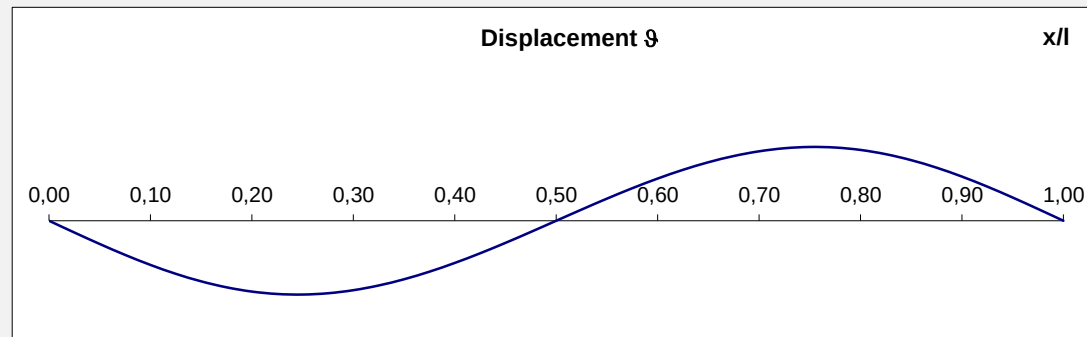
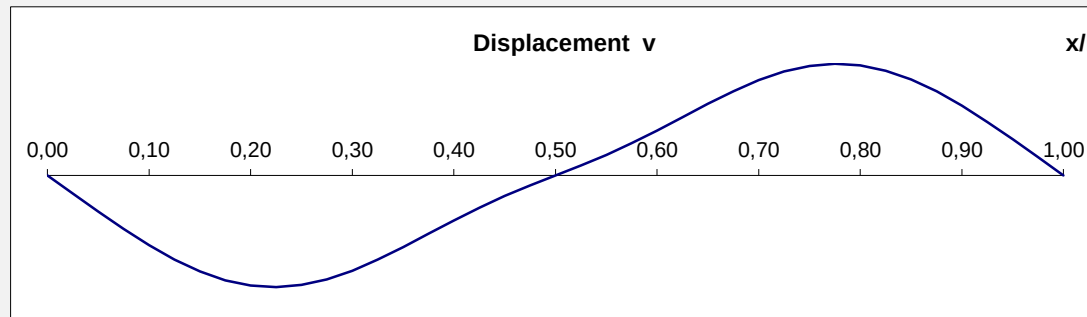
q_d : 150 kN/m

L : 6 m

f_y : 35.5 kN/cm²

3. Initial imperfections

- Determination of eigenmodes



3. Initial imperfections

- Calculation of initial imperfection

HEB 360 (Rolled profile)

$$\bar{\lambda}_{LT} = 0.703$$

$$h = 360 \text{ [cm]}$$

$$b = 300 \text{ [cm]}$$

$$h/b = 1.2 \leq 2.0$$

$$\text{Plastic analysis } e_0/L = 1/200 \rightarrow e_0 = 600/200 = 3 \text{ [cm]}$$

3. Initial imperfections

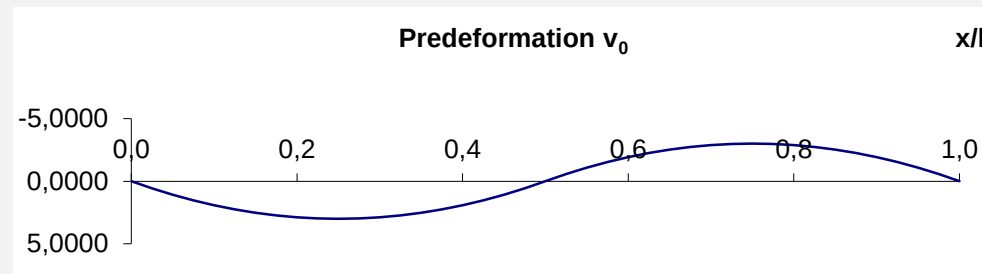
- Introduce imperfection into the software

Predeformations/

Type: **streight line + parabola**

Geometrical substitutional imperfections

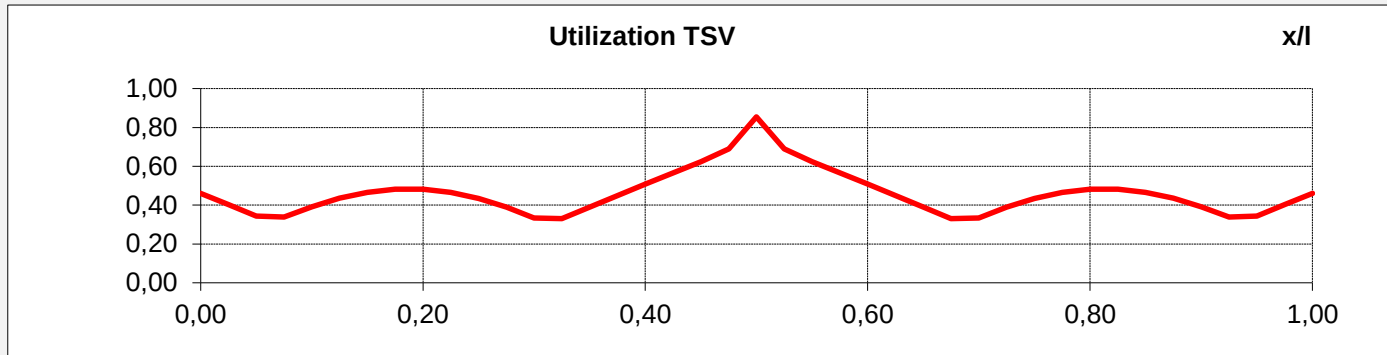
from x [cm]	to x [cm]	$w_{0,A}$ [cm]	$w_{0,E}$ [cm]	$w_{0,M}$ [cm]	$v_{0,A}$ [cm]	$v_{0,E}$ [cm]	$v_{0,M}$ [cm]	$\vartheta_{0,A}$ [rad]	$\vartheta_{0,E}$ [rad]	$\vartheta_{0,M}$ [rad]
0	600						3.00			
600	1,200						-3.00			



What happens with the internal forces?

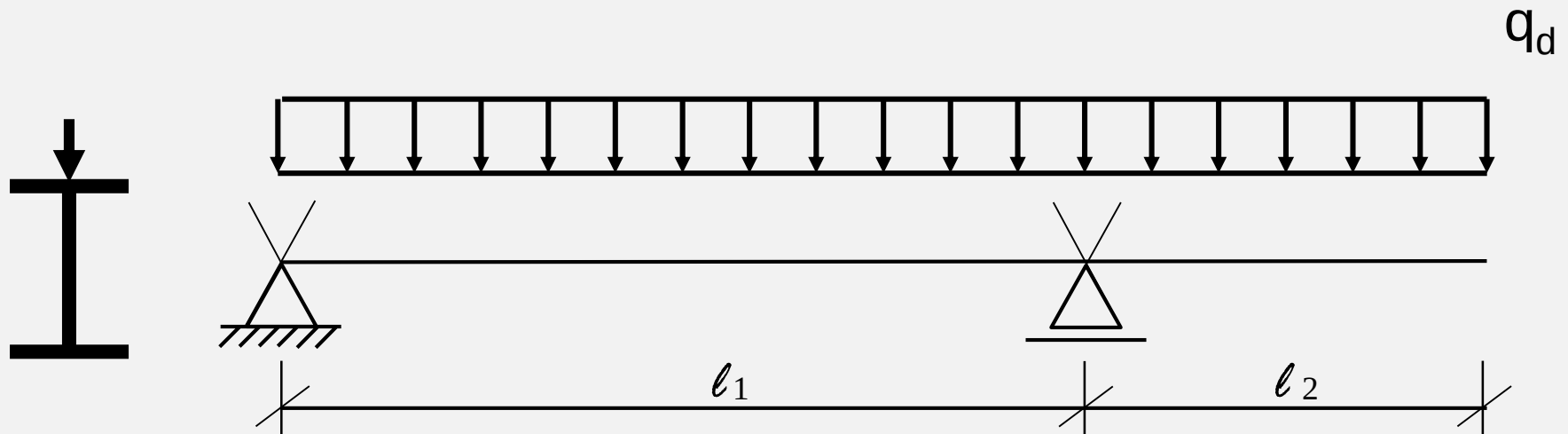
3. Initial imperfections

- Verification



max. utilization 0.855

4. More examples



L_1 : 1000 [cm]

f_y : 23.5

L_2 : 400 [cm]

q_d : 10 [kN/m]

IPE300

Distributed rotational spring : 9 [kN·m/m]

4. More examples

Span-1

IPE300 (rolled profile)

$h/b=300/150=2.0$

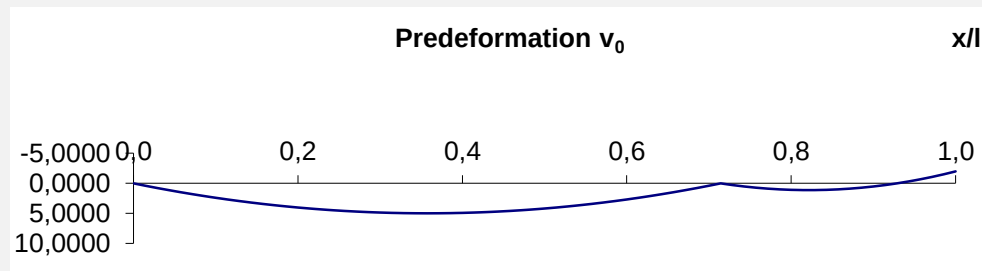
bow imperfection $e_0 = 1000/200 = 5$ [cm]

Span-2

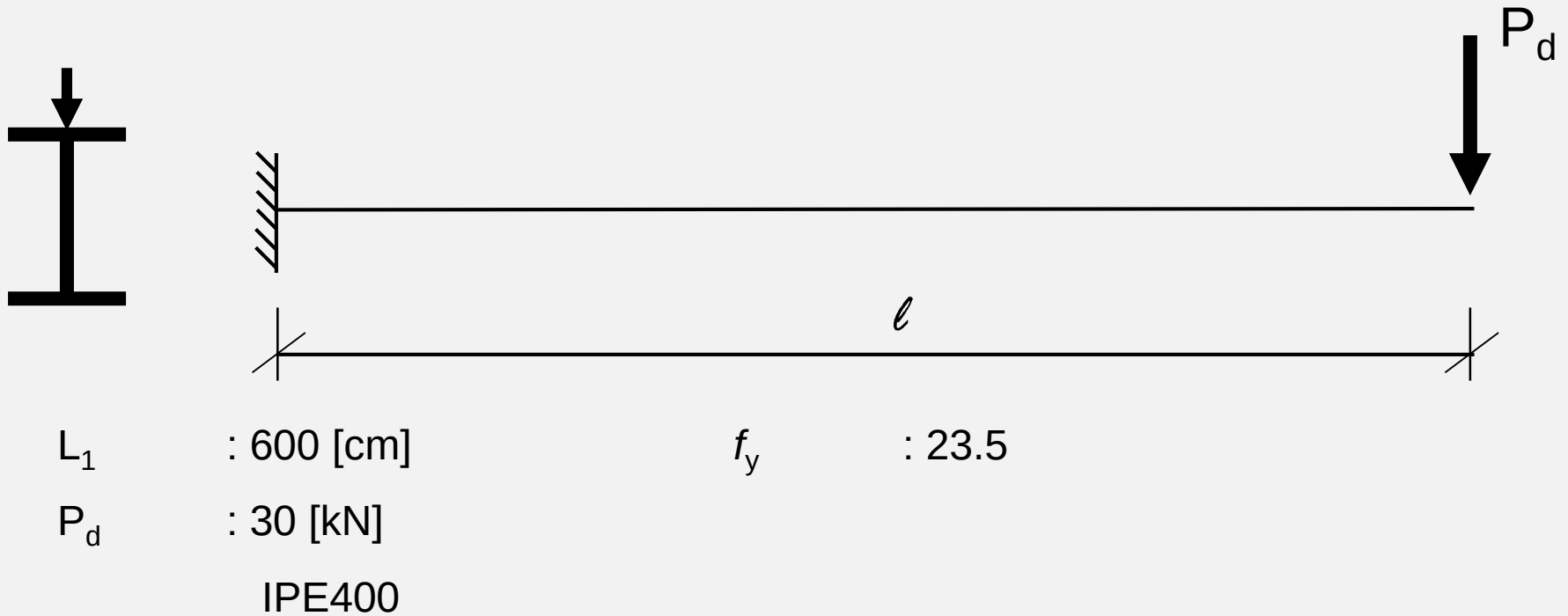
sway imperfection $e_0 = 400/200 = 2$ [cm]

bow imperfection $e_0 = 400/200 = 2$ [cm]

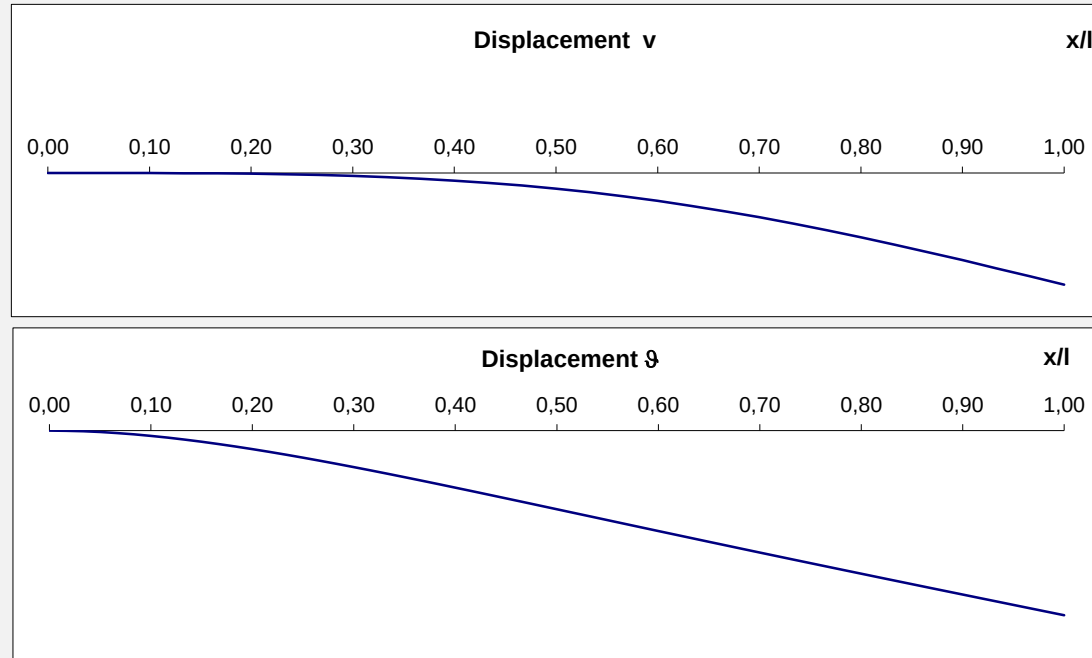
$$\lambda_{LT} = \sqrt{\frac{148}{113}} = 1.14$$



4. More examples



4. More examples

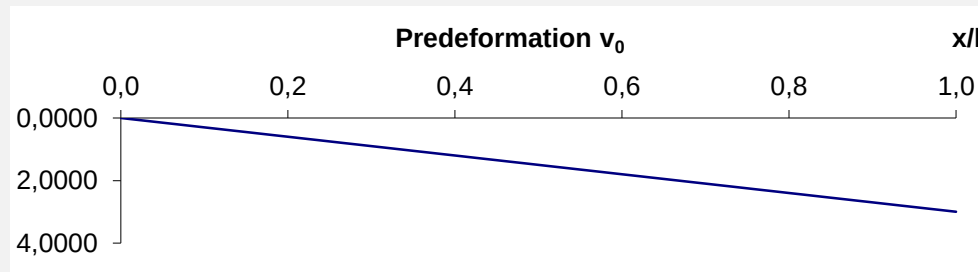


4. More examples

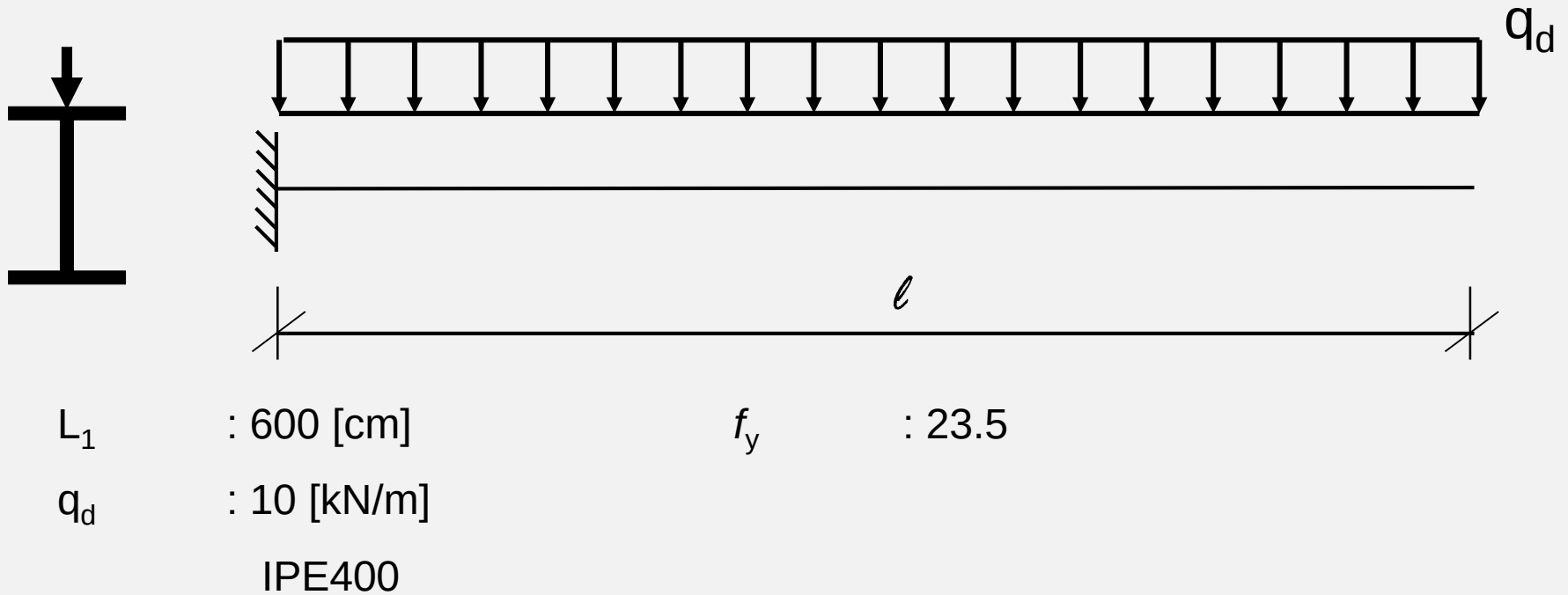
Span-1

IPE400 (rolled profile)

sway imperfection $e_0 = 600/200 = 3$ [cm]



4. More examples

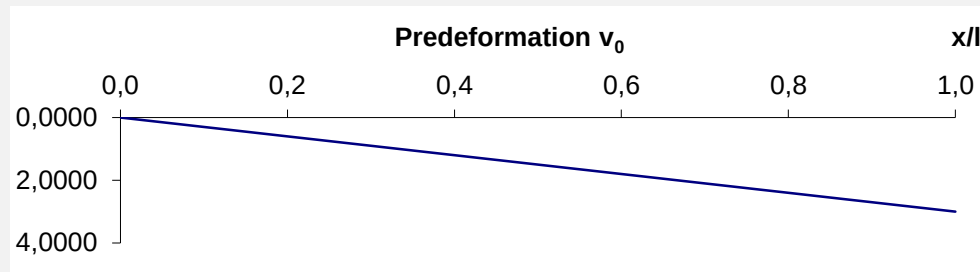


4. More examples

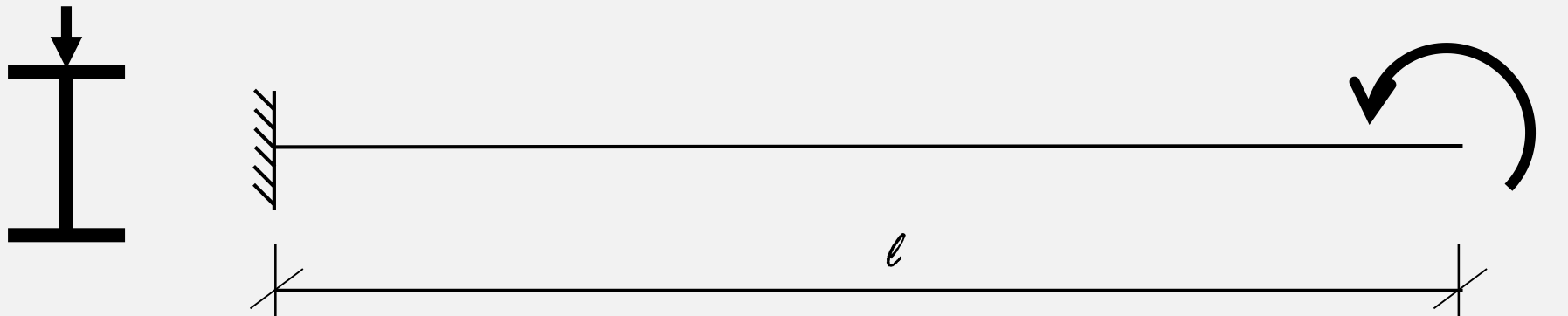
Span-1

IPE400 (rolled profile)

sway imperfection $e_0 = 600/200 = 3$ [cm]



4. More examples

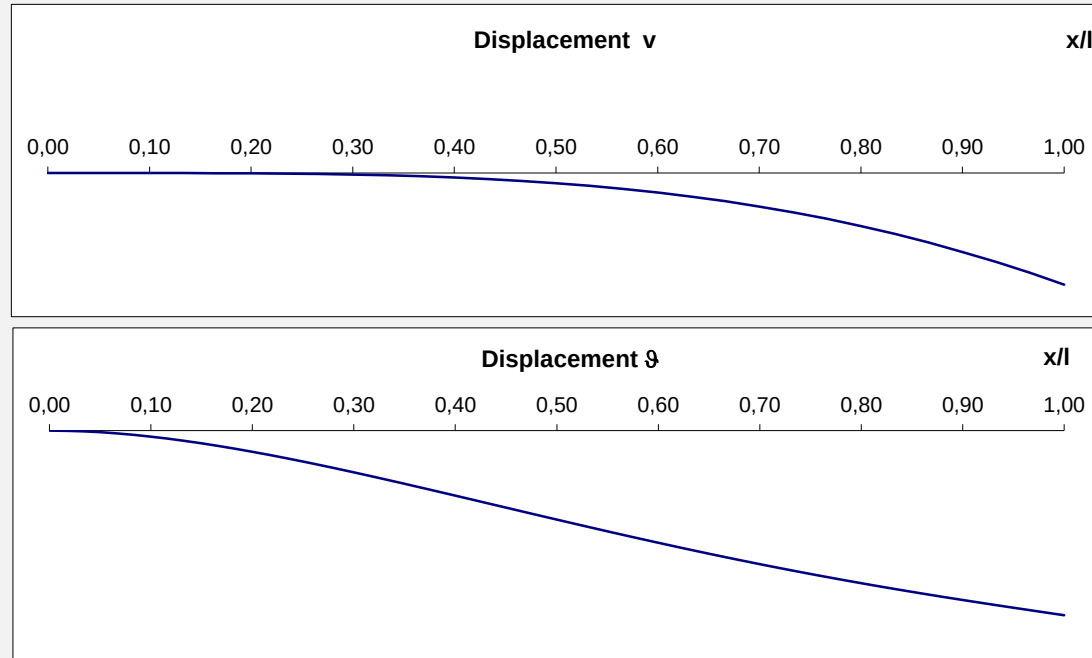


L_1 : 600 [cm] f_y : 23.5

M_d : 600*30 [kN cm]

IPE400

4. More examples



References

- [1] Kindmann, R., Kraus, M., 2012. Steel Structures: Design using FEM. Ernst & Sohn Verlag für Architektur und technische Wissenschaften GmbH & Co. KG, Berlin, Germany.
- [2] Comité Européen de Normalisation (CEN), 2005. Eurocode 3: Design of steel structures - Part 1-1: General rules and rules for buildings.



Bauhaus-Universität Weimar

Professur Stahl- und Hybridbau

Vielen Dank ...