Wind-induced Vibrations of Structures

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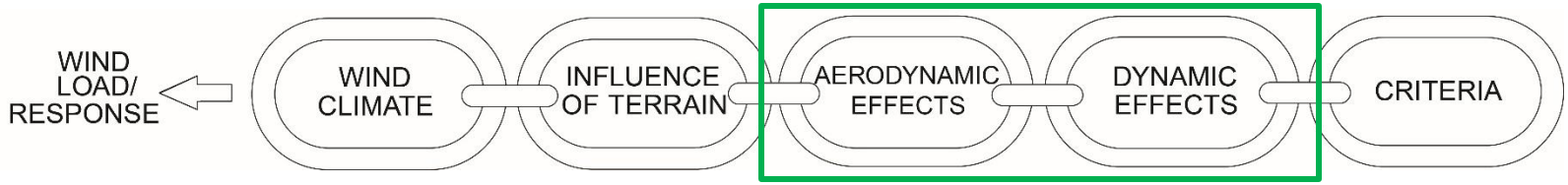
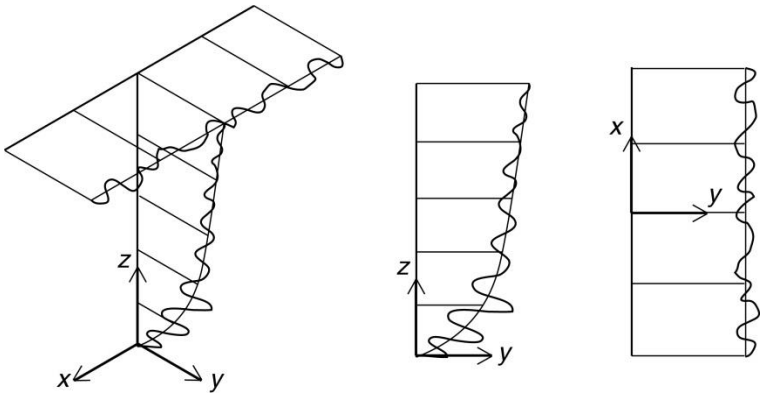
Motivation

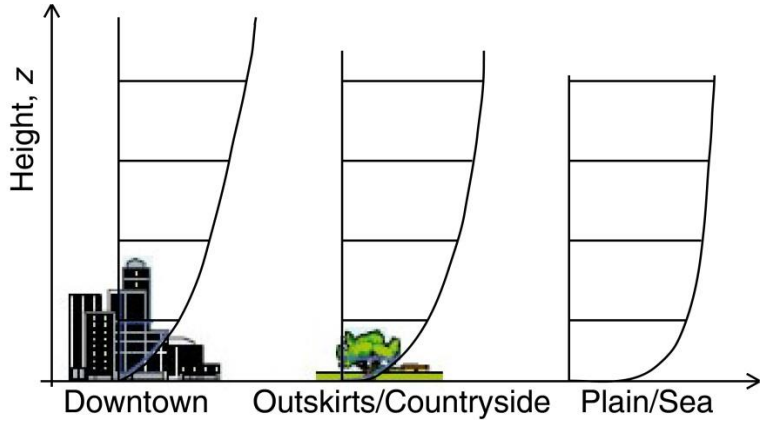
- Long and slender structures
- Light weight
- Low structural damping

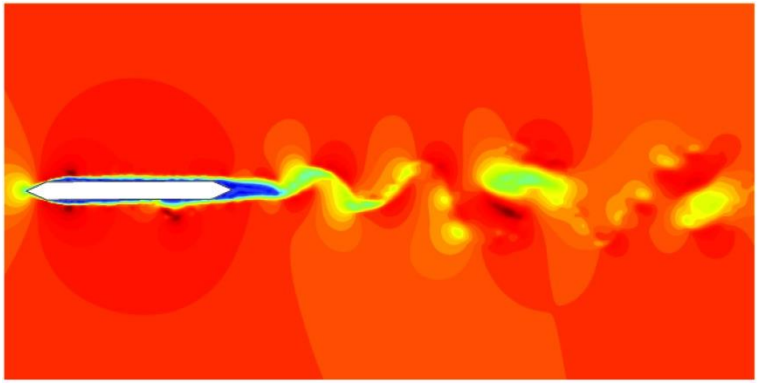
Wind Engineering (Design)

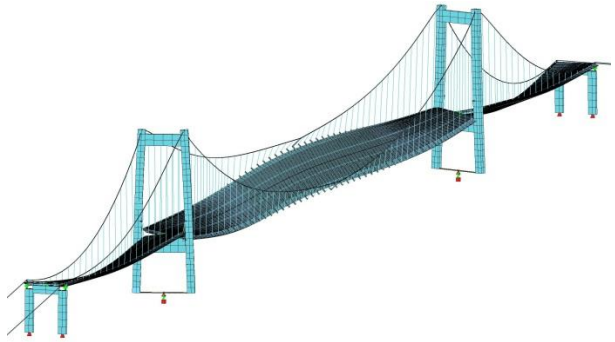


[Alan G. Davenport Wind Loading Chain]

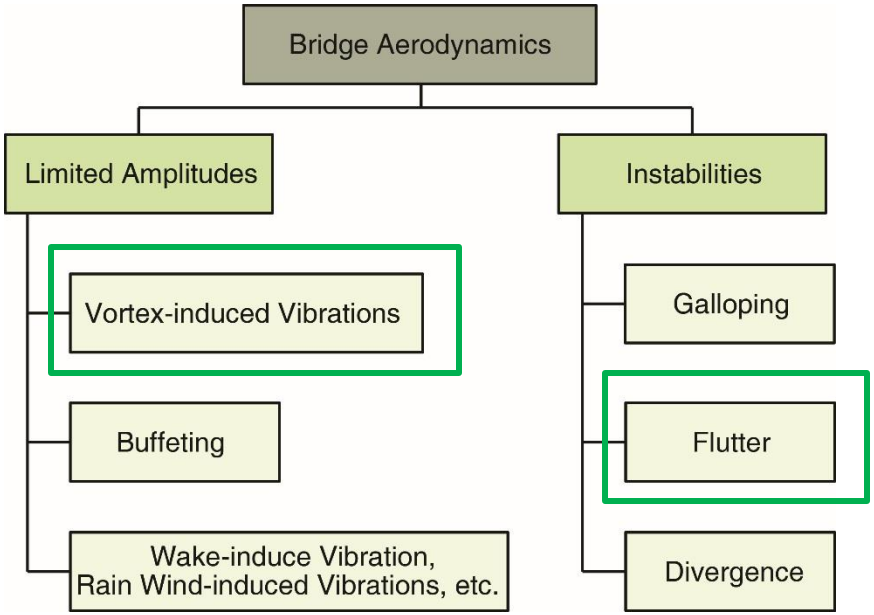


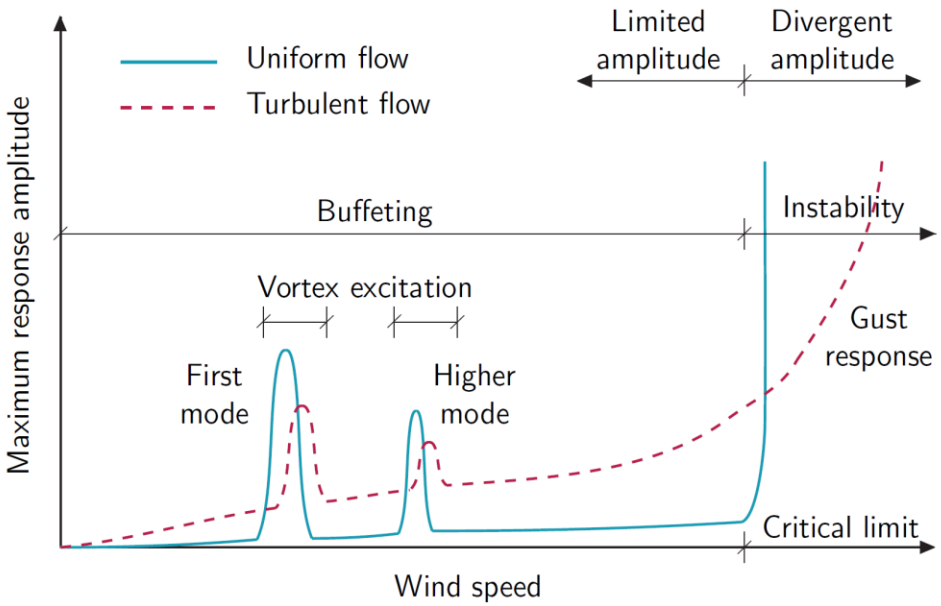






Aerodynamic Phenomena



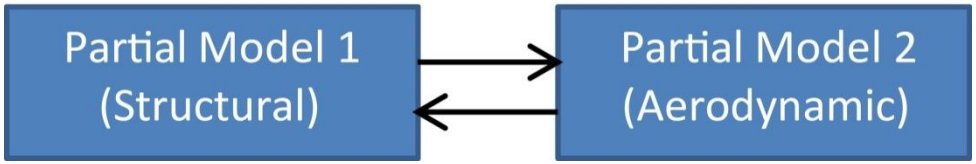


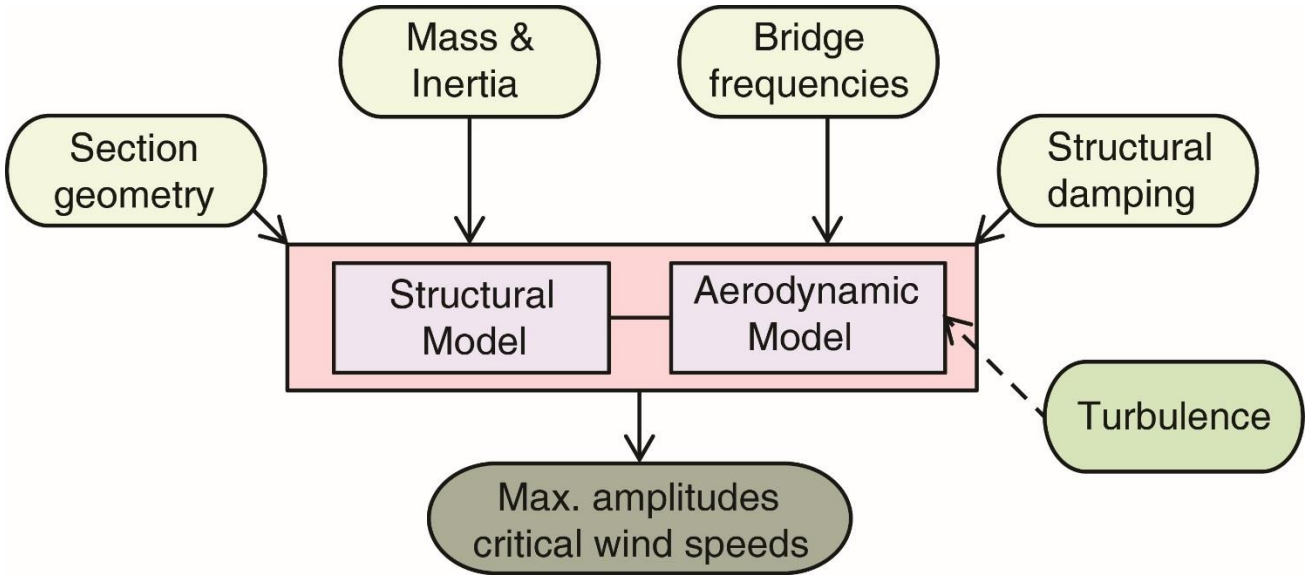
Aerodynamic Forcing

1. Static mean forces
2. Vortex shedding forces
3. Motion-induced forces
4. Buffeting forces

Fluid-structure Interaction

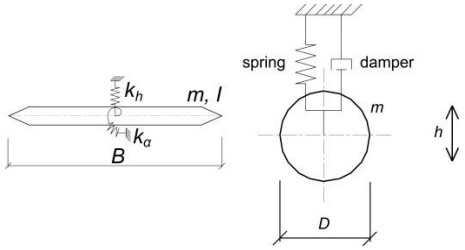
Coupling of aerodynamic forcing with a structural dynamics problem

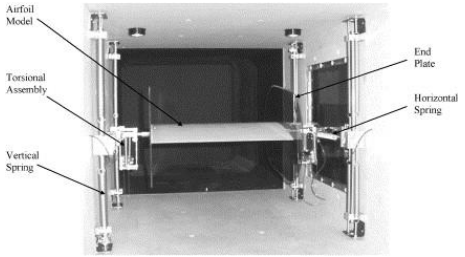




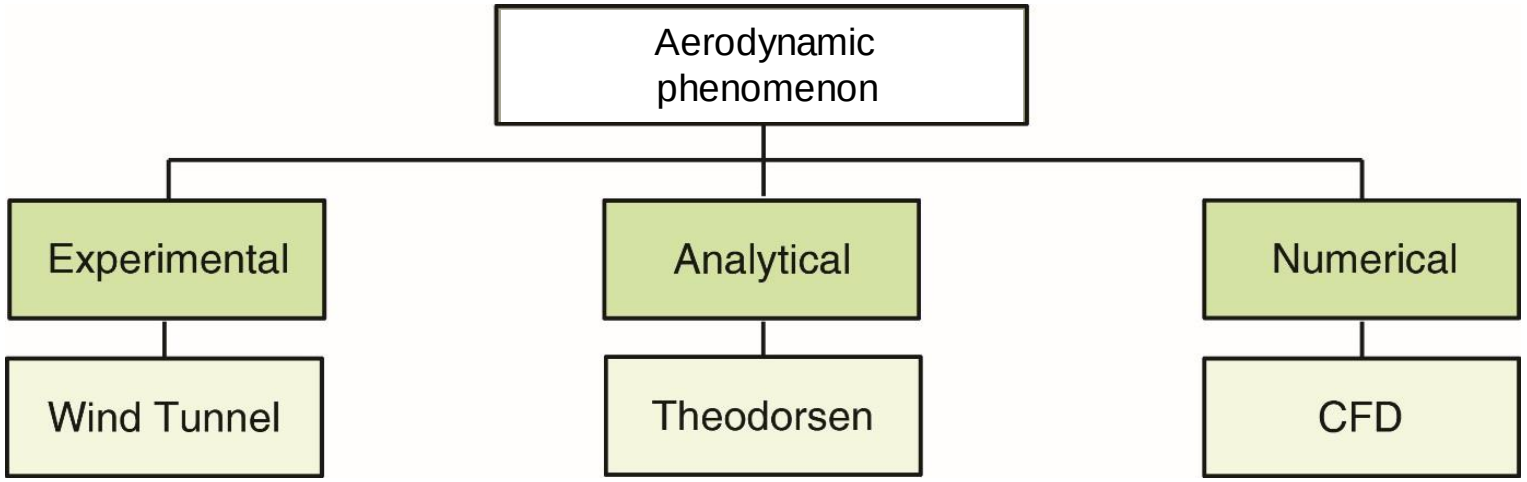
Model Assumptions

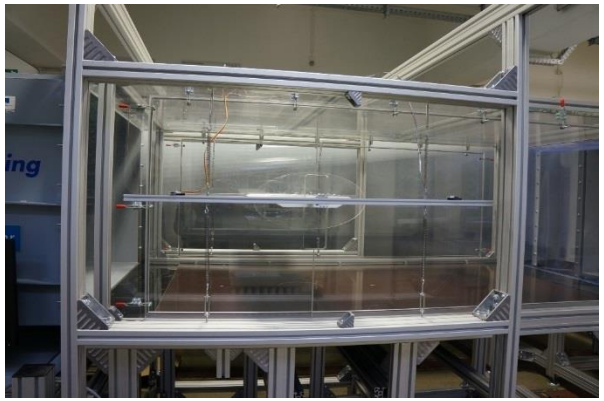
- FE model with solid, shell or beam elements
- Dimensional reduction from 3D to 2D
- Modal damping or Rayleigh damping
- Number of modes to be considered
- Linear system

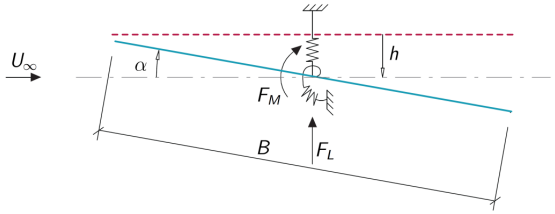


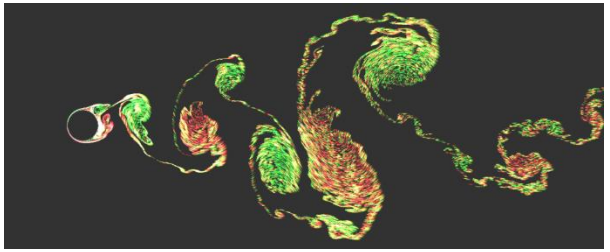


Methods of Aerodynamic Analysis





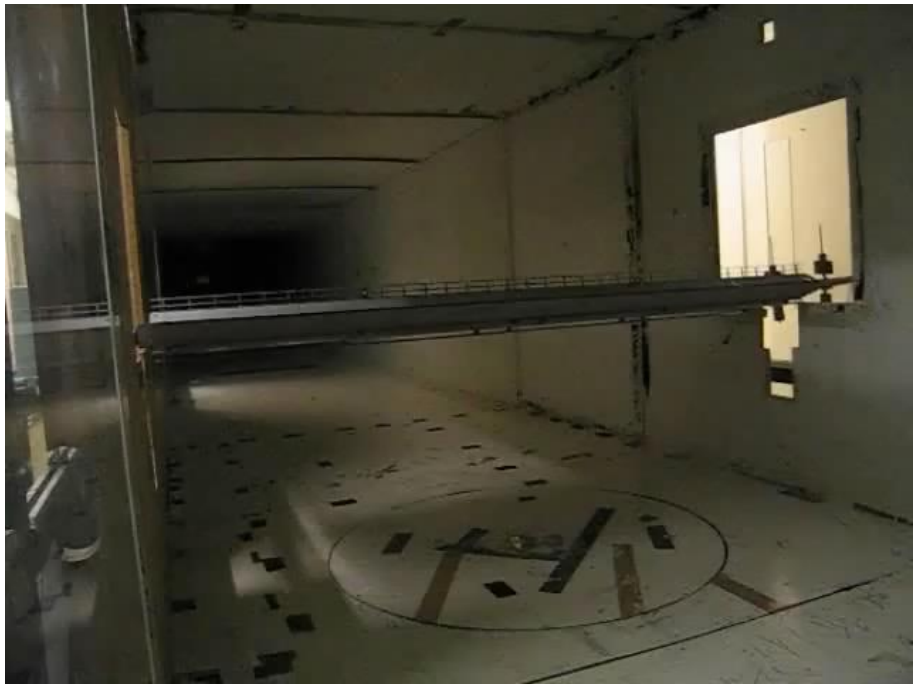




Experimental Methods

Wind Tunnel Tests

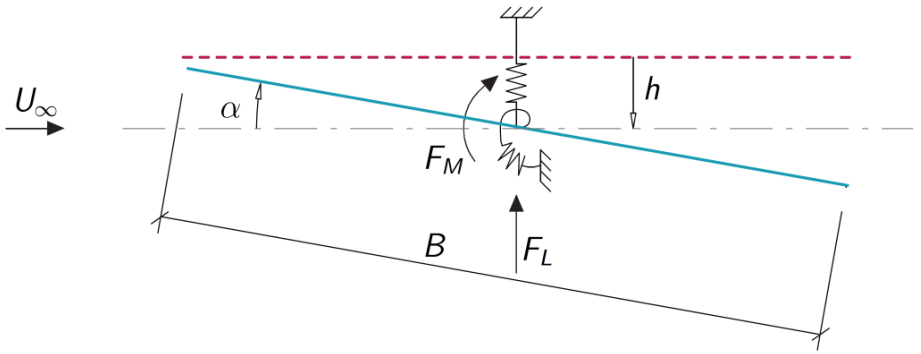
- Section model tests
- 3D model tests



Analytical Methods

Theodorsen Theory

- Based on potential flow theory
- Only for flat plate or airfoils
- Only for 2D flow



$$F_L = -\rho b^2 U_\infty \pi \dot{\alpha} - \rho b^2 \pi \ddot{h} - 2\pi\rho C U_\infty^2 b\alpha - 2\pi\rho C U_\infty b\dot{h} - 2\pi\rho C U_\infty b^2 \frac{1}{2}\dot{\alpha}$$

$$F_M = -\rho b^2 \pi \frac{1}{2} U_\infty b\dot{\alpha} - \rho b^4 \pi \frac{1}{8}\ddot{\alpha} + 2\rho U_\infty b^2 \pi \frac{1}{2} C U_\infty \alpha + 2\rho U_\infty b^2 \pi C\dot{h} + 2\rho \frac{1}{2} U_\infty b^3 \pi C\dot{\alpha}$$

$$k = \frac{b\omega}{U} = \frac{K}{2}$$

Numerical Methods

Finite Element Method (FEM)

Finite Volume Method (FVM)

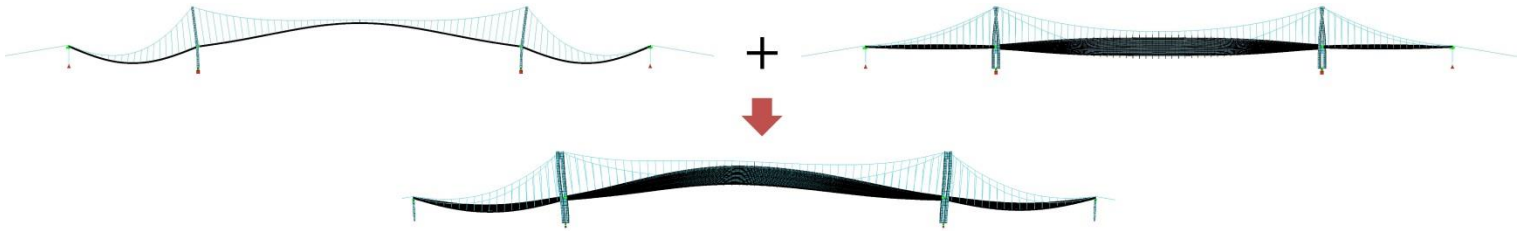
Vortex Particle Method (VPM)

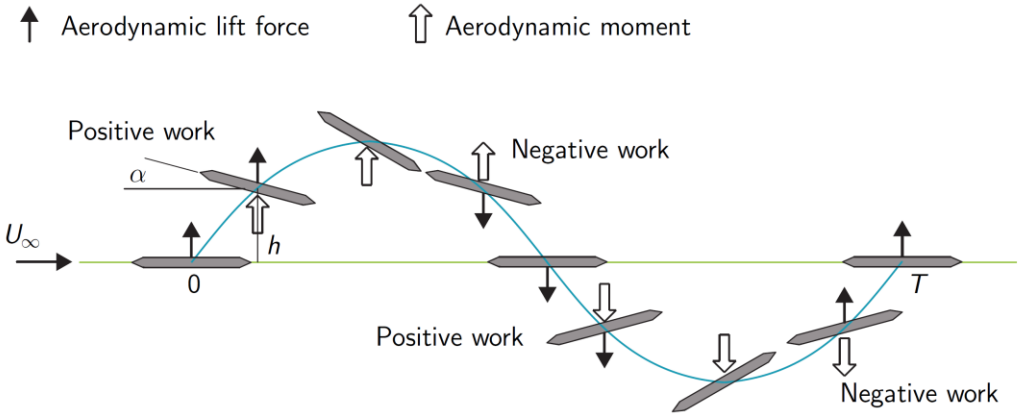
- Solution of Navier-Stokes equation with particles
- Grid free method
- Only for 2D flow
- Uniform incident flow

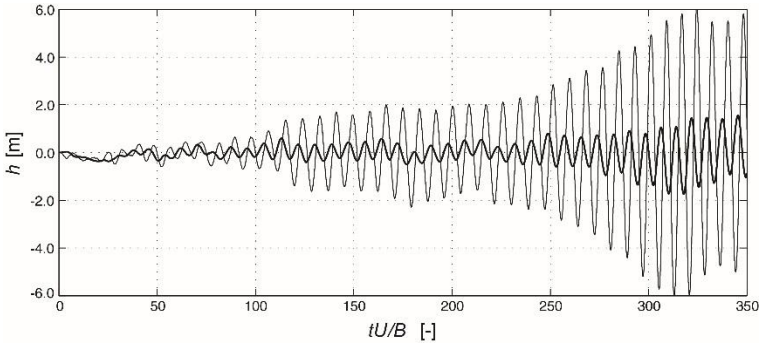
Classical Flutter

Classical Flutter is an aeroelastic phenomenon in which two degrees of freedom of a structure, rotation and vertical translation, couple together in a flow-driven, unstable oscillation

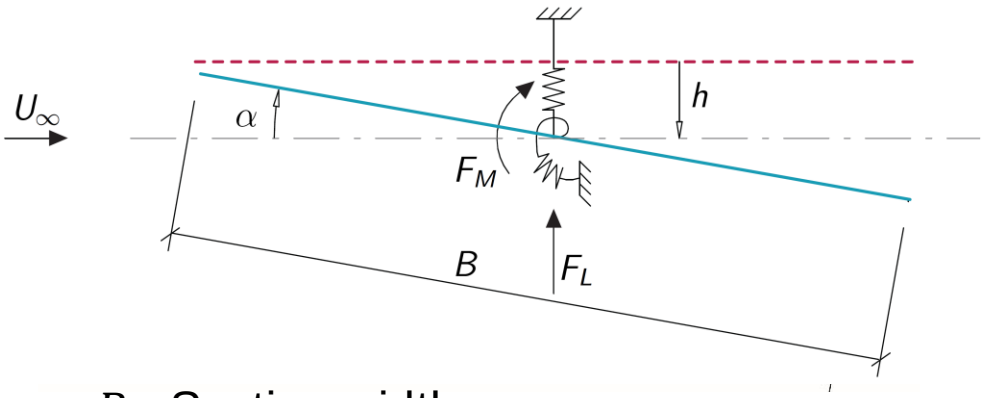
[Simiu and Scanlan, 1996]







Scanlan Meta Model and Derivatives

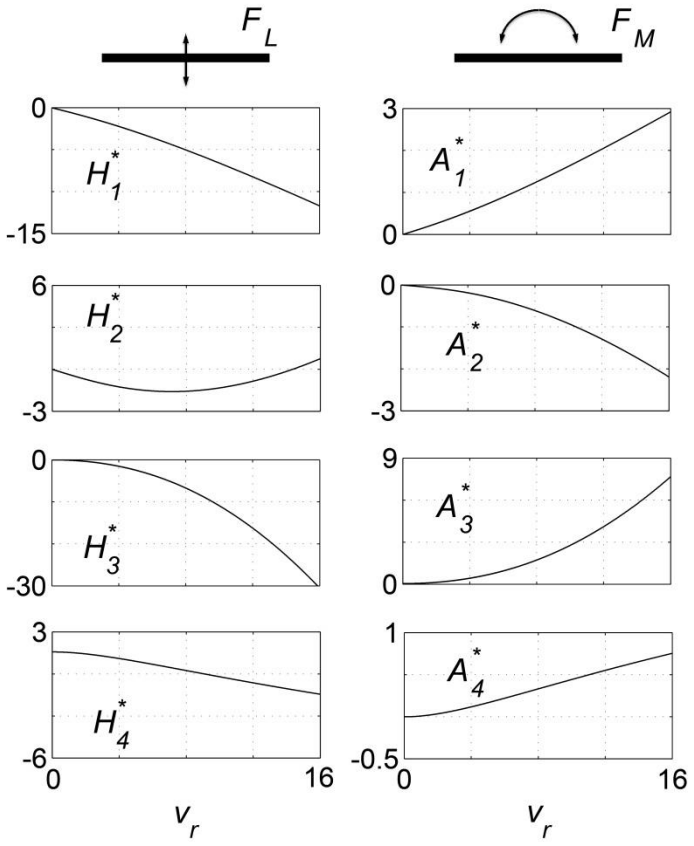


B - Section width
U_∞ - Oncoming wind speed
h - Vertical displacement
α - Rotation

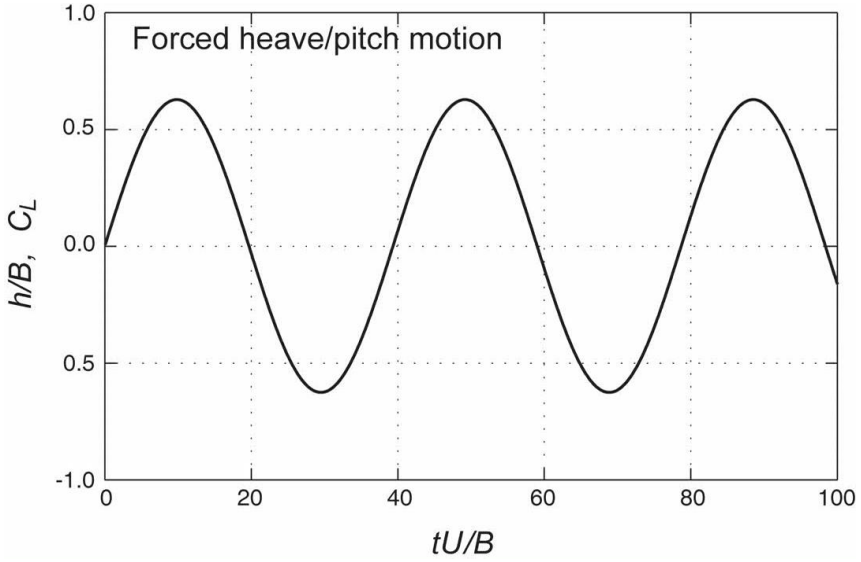
$$m\ddot{h} + 2m\xi_h\omega_h\dot{h} + m\omega_h^2 h = F_L$$
$$I\ddot{\alpha} + 2I\xi_\alpha\omega_\alpha\dot{\alpha} + I\omega_\alpha^2\alpha = F_M$$

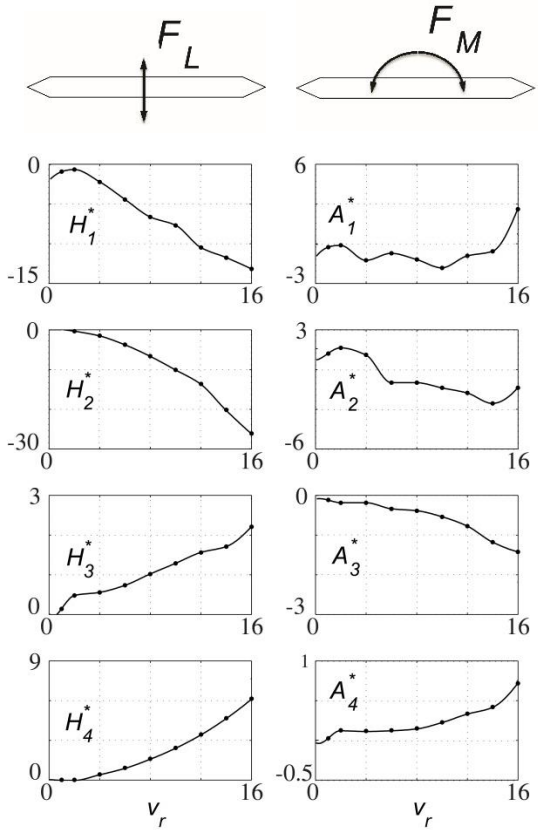
$$v_r = \frac{2\pi U_\infty}{\omega B} = \frac{2\pi}{K}$$
$$F_L = \tfrac{1}{2}\rho U_\infty^2 B\left[KH_1^*\frac{\dot{h}}{U_\infty} + KH_2^*\frac{B\dot{\alpha}}{U_\infty} + K^2H_3^*\alpha + K^2H_4^*\frac{h}{B}\right]$$
$$F_M = \tfrac{1}{2}\rho U_\infty^2 B^2\left[KA_1^*\frac{\dot{h}}{U_\infty} + KA_2^*\frac{B\dot{\alpha}}{U_\infty} + K^2A_3^*\alpha + K^2A_4^*\frac{h}{B}\right]$$

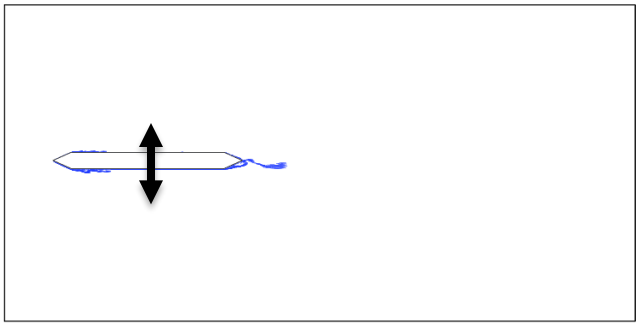
$H_i^*(\omega)$, $A_i^*(\omega)$ - Aerodynamic Derivatives



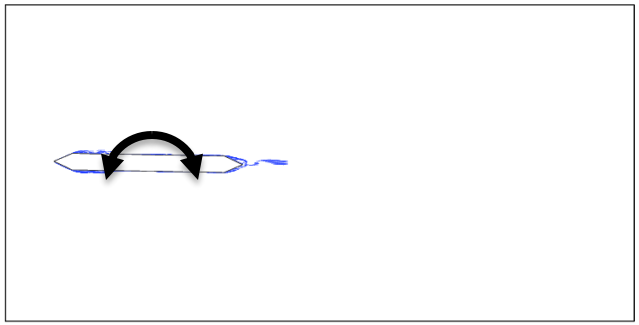
Determination of Aerodynamic Derivatives







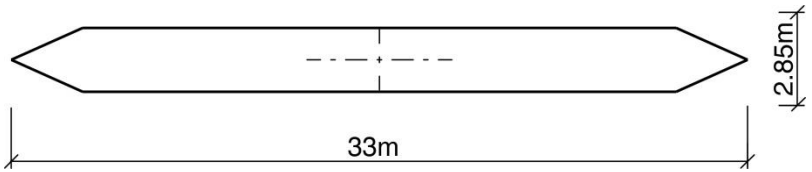
Heave motion

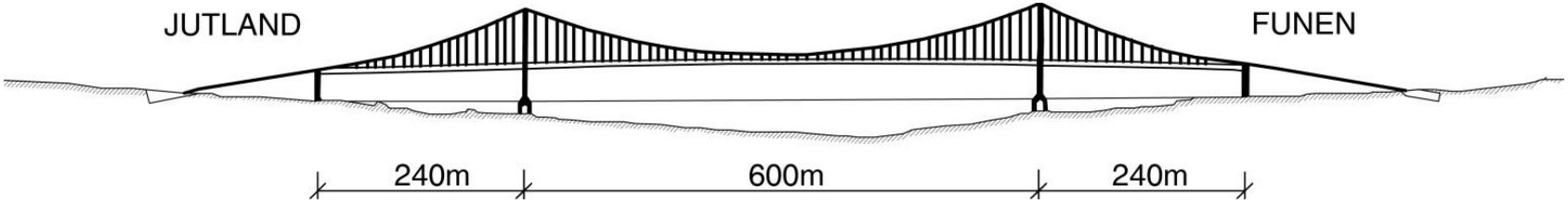


Pitch motion

Reference Object

Lillebælt Suspension Bridge Denmark



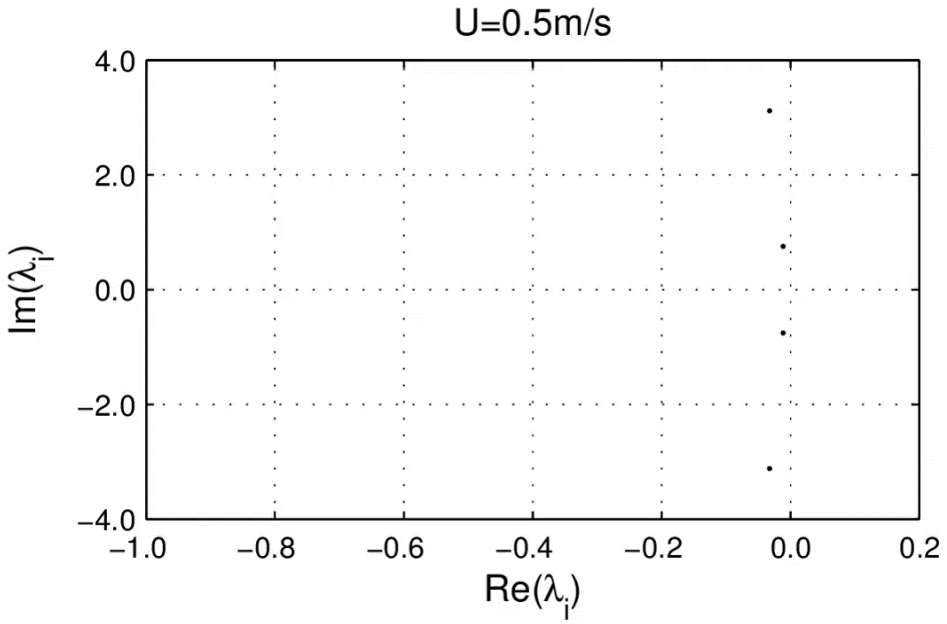


Section width [m]	Mass [kg m^{-1}]	Inertial mass [kg-m^2 m^{-1}]	Bending frequency [Hz]	Torsional frequency [Hz]	Damping ratio [-]
33	11,667	10,17,778	0.156	0.500	0.01

Derivative-based Eigenvalue Analysis

- ✓ Frequency domain
- ✓ Simple and easy to use and better insight into the force coupling
- ✓ Independent of the shape of the body
- ✓ Considers the behaviour of real cross section

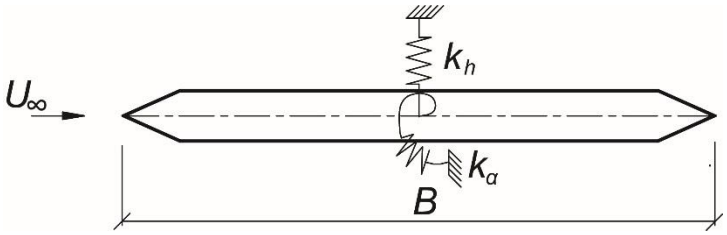
- ✗ Ignores effects from dimensional reduction to 2-DOF
- ✗ Prior knowledge of mode coupling



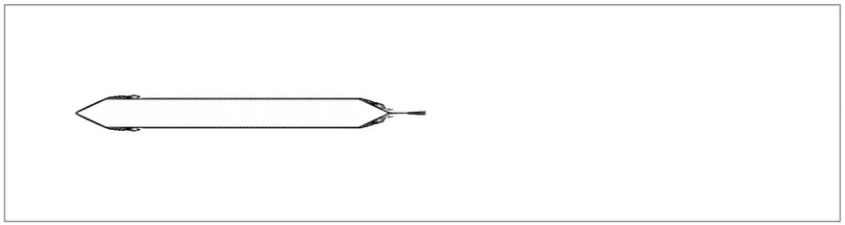
Fluid-structure Interaction (dynamic) Analysis

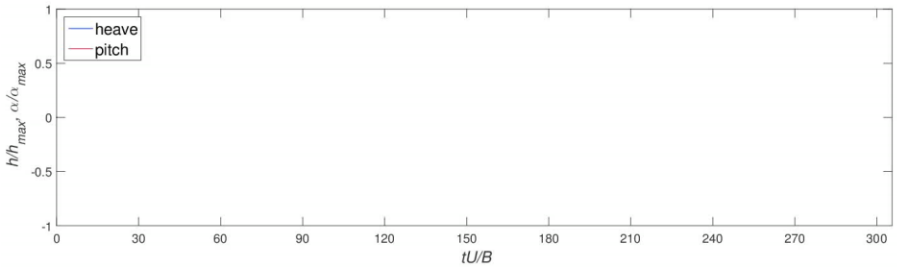
- ✓ Better representation of the phenomenon
- ✓ Shape of the section

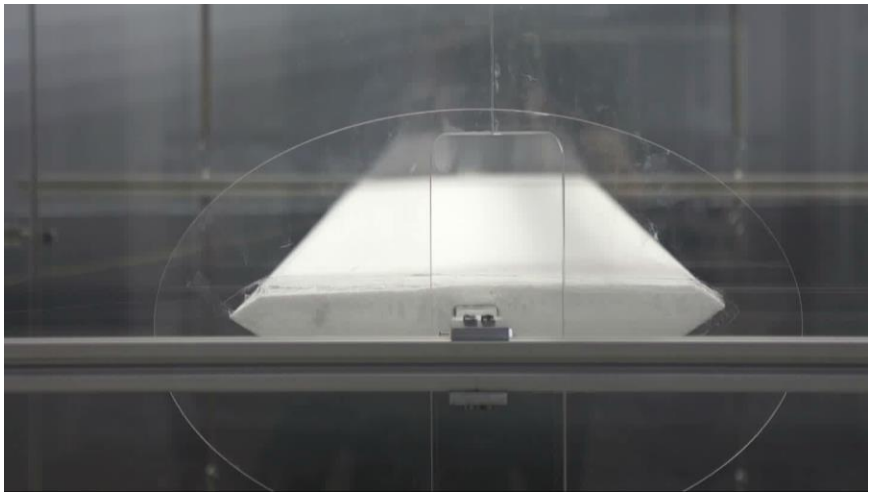
- ✗ Dimensional reduction to two modes only
- ✗ Model for 2D wind only
- ✗ Not for non-prismatic sections



U = 95 m/s



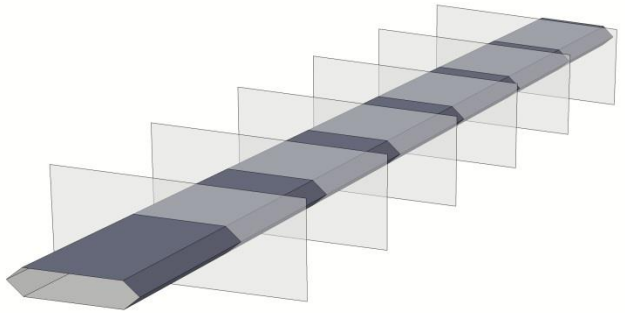




Scale 1:100

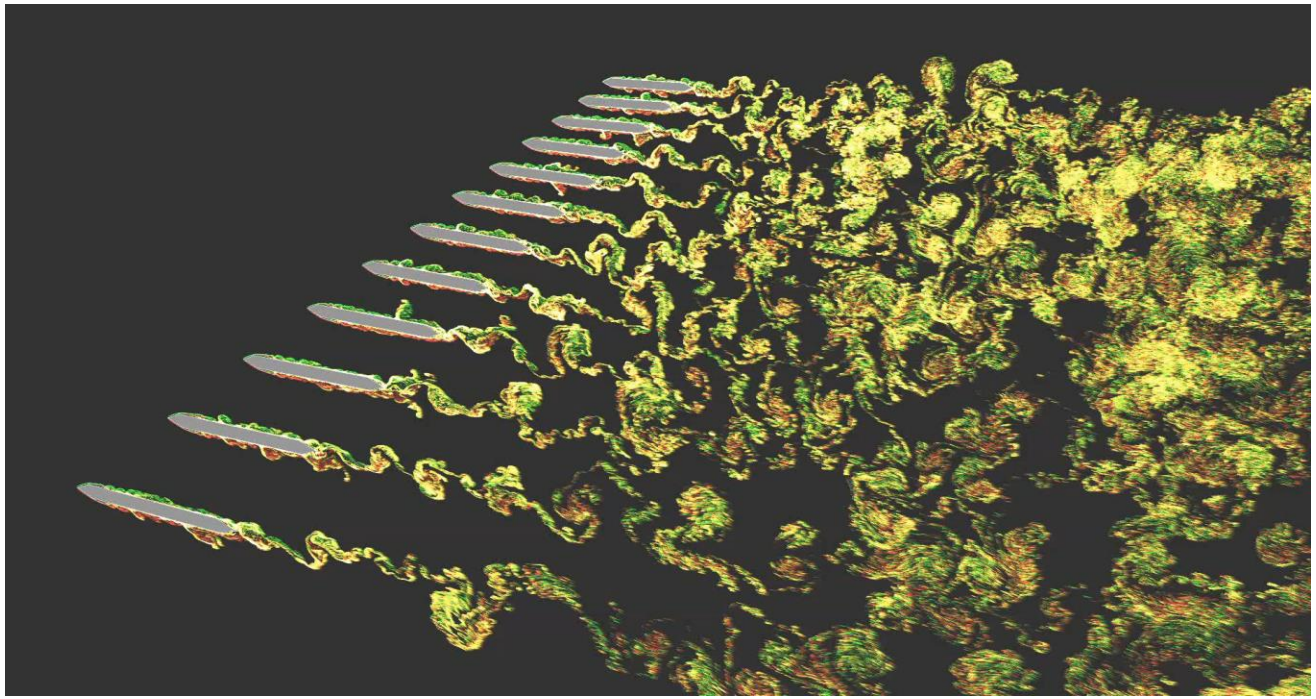
Quasi-3D Approach (Multislice Approach)

- ✓ Applicable to line like structures
- ✓ Freedom to choose number of modes
- ✓ Modal damping can be used
- ✓ No prior knowledge of mode coupling

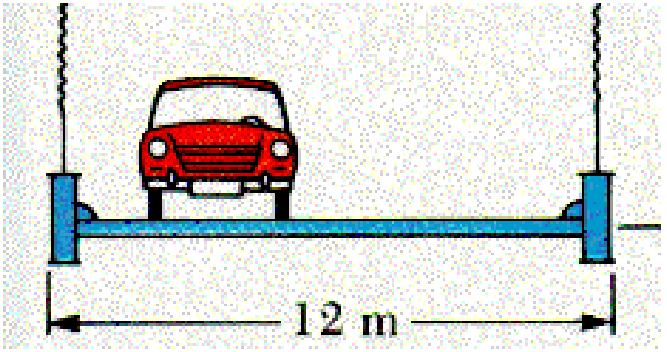


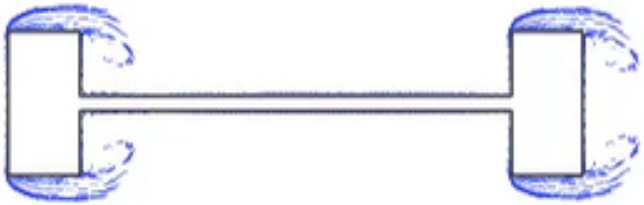
- ✗ No correlation between aerodynamic forces

U = 95 m/s



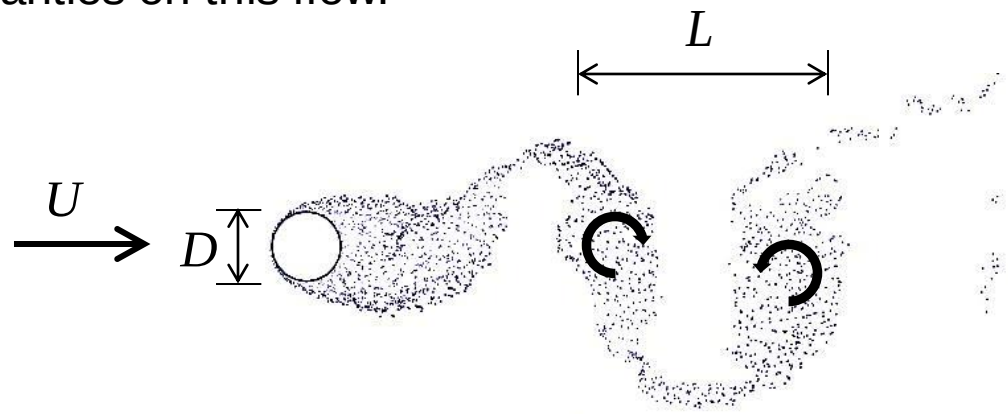
Vortex Shedding





Vortex Induced Vibrations (VIV)

Vortex-induced vibrations are motions induced on bodies facing an external flow by periodical irregularities on this flow.



Structural natural frequency

$$f_n = (m, k)$$

Vortex shedding frequency

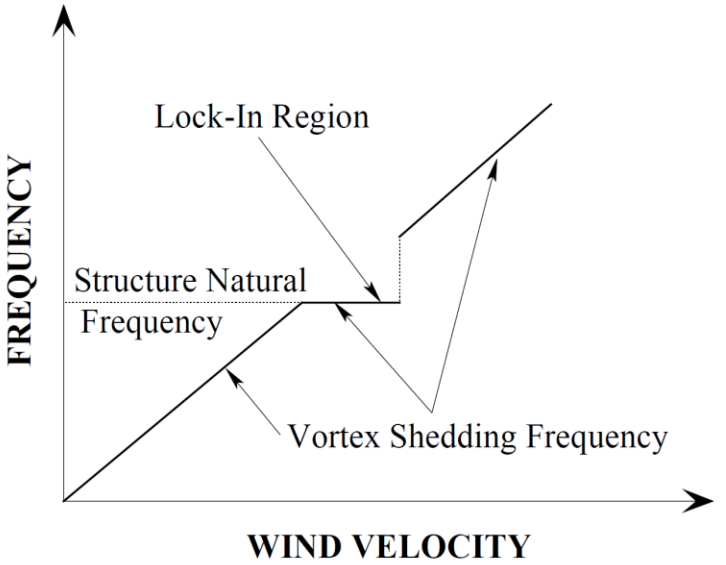
$$f_v = f(D, U, shape)$$

$$f_v = \frac{U}{L}$$

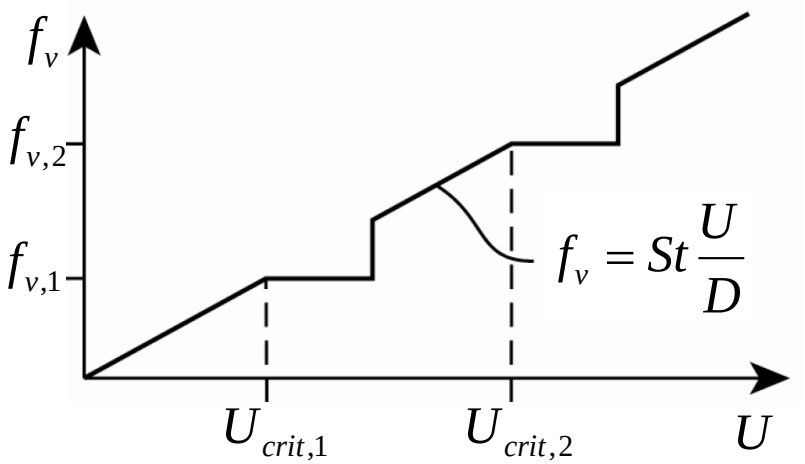
Lock-in → Resonance

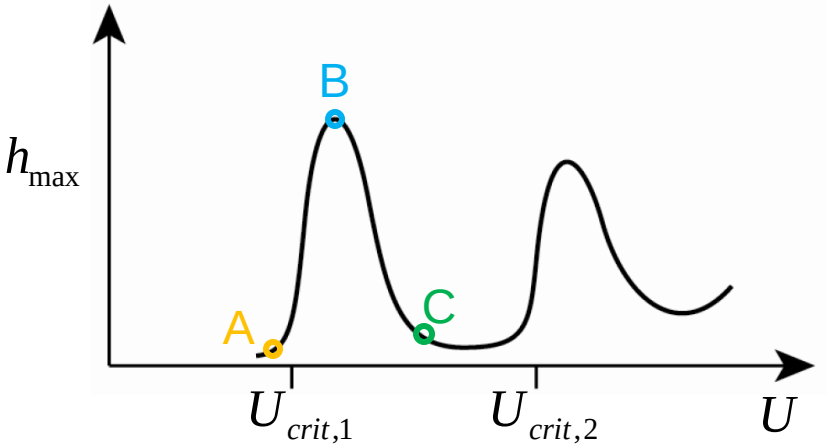
$$f_v = (0.8 - 1.2)\, f_n$$

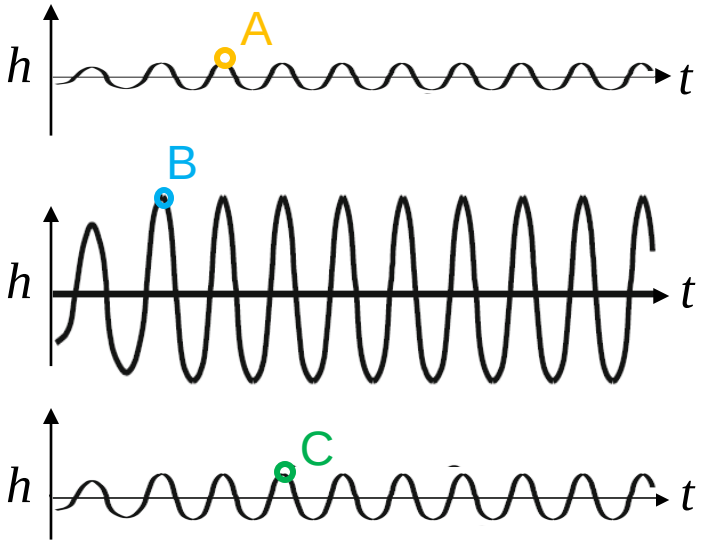
Self-limiting phenomenon, potentially destructive



Simulating Sensor System Attributes







Vortex Induced Vibrations

- Strouhal number

$$St = \frac{f_v D}{U}$$

- Reynolds number

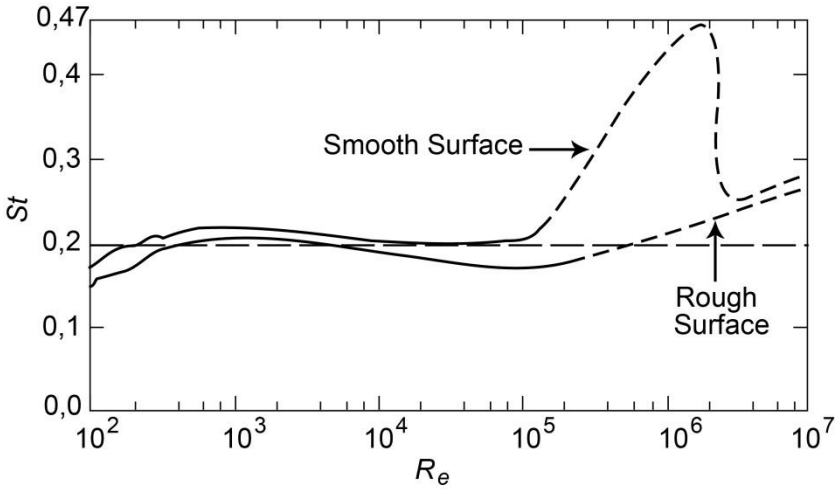
$$R_e = \frac{UD}{\nu}$$

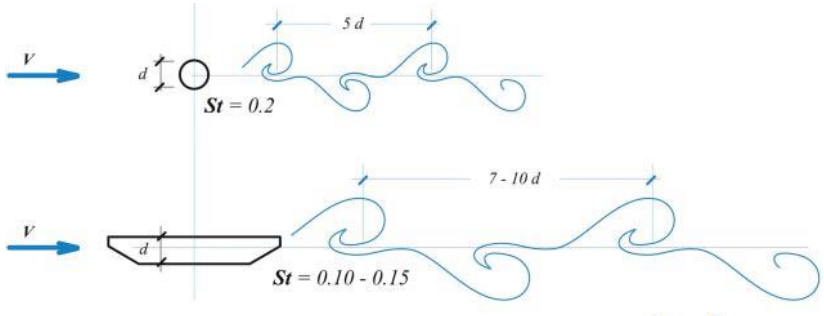
- Scruton number

$$Sc = \frac{\xi m}{\rho D^2}$$

- Coefficient of drag

$$C_D = \frac{F_D}{\frac{1}{2}\rho U^2 D}$$





ν - kinematic viscosity

ξ - damping ratio to the critical,

m - mass per unit length and

ρ - air density

F_D - drag force on the cylinder

Estimation of oscillation amplitudes

- Eurocode (EN 1991 1-4) [Harmonic model, Ruscheweyh, 1990]

$$\frac{y_{max}}{D} = \frac{1}{St^2} \cdot \frac{1}{Sc} \cdot K \cdot K_w \cdot C_{lat}$$

- Canadian Standard (CAN/CSA-S6-06) [1DOF-model, Vickery & Basu]

$$a_i = \frac{3.5\,\tilde{C}_L D^4 \pi^{0.25} C}{\sqrt{B\xi_i}(4\pi S)^2 G M_i},\ y_i(x) \le 0.025D\,;\ a_i = \frac{\sqrt{2}\tilde{C}_L D^3 \int_0^H |\mu_i(x)|dx}{(4\pi S)^2 \xi_i G M_i},\ y_i(x) \ge 0.025D$$

- British Standard (BD 49/01) [Harmonic model, bridges]

$$y_{max} = \frac{c b^{0.5} d_4^{2.5} \rho}{4 m \delta_s};\ \rightarrow \frac{y_{max}}{d_4} = \frac{1}{Sc} \cdot \sqrt{\frac{b}{d_4}} \cdot c$$

- Nonlinear model

[amplitude-modulated delay-time quasi-static hypothesis, A. Barrero]

$$r \approx \frac{U_{r0}^2}{4Sc}\left(\frac{|a_1|}{U_{r0}} + C_{YT}\right)\cos(r)$$

EN 1991-1-4:2005, Eurocode 1: Actions on structures Part 1-4: General actions — Wind actions

E.1.5.2.1 Calculation of displacements $y_{F,max}$

The largest displacement $y_{F,max}$ can be calculated using Expression (E.7).

$$\frac{y_{F,max}}{b} = \frac{1}{St^2} \cdot \frac{1}{Sc} \cdot K \cdot K_W \cdot c_{lat} \qquad \text{(E.7)}$$

where:

St is the Strouhal number given in Table E.1

Sc is the Scruton number given in E.1.3.3

K_W is the effective correlation length factor given in E.1.5.2.4

K is the mode shape factor given in E.1.5.2.5

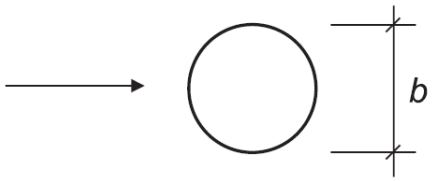
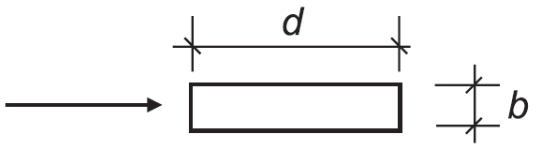
c_{lat} is the lateral force coefficient given in Table E.2

NOTE The aeroelastic forces are taken into account by the effective correlation length factor K_W.

E.1.3.2 Strouhal number St

The Strouhal number *St* for different cross-sections may be taken from Table E.1.

Table E.1 — Strouhal numbers *St* for different cross-sections

Cross-section	*St*
 for all *Re*-numbers	0,18
 $0{,}5 \leq d/b \leq 10$	from Figure E.1

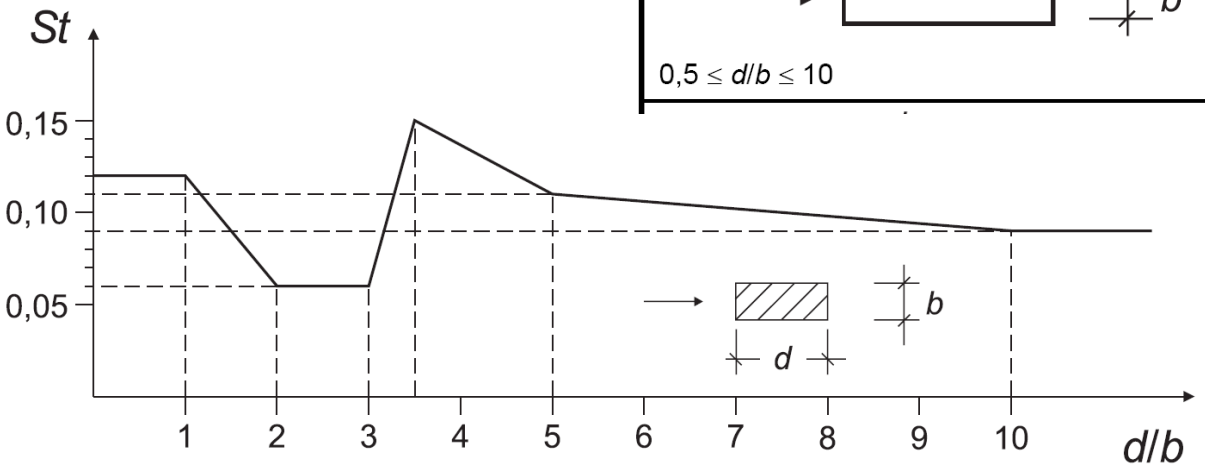


Figure E.1 — Strouhal number (*St*) for rectangular cross-sections with sharp corners

E.1.3.3 Scruton number Sc

(1) The susceptibility of vibrations depends on the structural damping and the ratio of structural mass to fluid mass. This is expressed by the Scruton number *Sc*, which is given in Expression (E.4).

$$Sc = \frac{2 \cdot \delta_s \cdot m_{i,e}}{\rho \cdot b^2} \qquad \text{(E.4)}$$

where:

δ_s is the structural damping expressed by the logarithmic decrement.

ρ is the air density under vortex shedding conditions.

$m_{i,e}$ is the equivalent mass m_e per unit length for mode *i* as defined in F.4 (1)

b is the reference width of the cross-section at which resonant vortex shedding occurs

Table F.2 —Approximate values of logarithmic decrement of structural damping in the fundamental mode, δ_s

Structural type	**structural damping, δ_s**
reinforced concrete buildings	0,10
steel buildings	0,05

F.4 Equivalent mass m_e

(1) The equivalent mass per unit length m_e of the fundamental mode is given by Expression (F.14).

$$m_e = \frac{\int_0^{\ell} m(s) \cdot \Phi_1^2(s)\, ds}{\int_0^{\ell} \Phi_1^2(s)\, ds} \qquad (F.14)$$

where:

m is the mass per unit length

ℓ is the height or span of the structure or the structural element

i = 1 is the mode number

(2) For cantilevered structures with a varying mass distribution m_e may be approximated by the average value of m over the upper third of the structure h_3 (see Figure F.1).

(3) For structures supported at both ends of span ℓ with a varying distribution of the mass per unit length m_e may be approximated by the average value of m over a length of $\ell/3$ centred at the point in the structure in which $\Phi(s)$ is maximum (see Table F.1).

E.1.3.1 Critical wind velocity v_{crit}

(1) The critical wind velocity for bending vibration mode *i* is defined as the wind velocity at which the frequency of vortex shedding equals a natural frequency of the structure or a structural element and is given in Expression (E.2).

$$v_{crit,i} = \frac{b \cdot n_{i,y}}{St} \tag{E.2}$$

where:

b is the reference width of the cross-section at which resonant vortex shedding occurs and where the modal deflection is maximum for the structure or structural part considered; for circular cylinders the reference width is the outer diameter

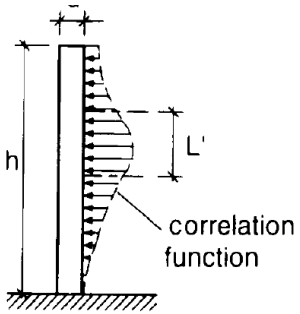
$n_{i,y}$ is the natural frequency of the considered flexural mode *i* of cross-wind vibration; approximations for $n_{1,y}$ are given in F.2

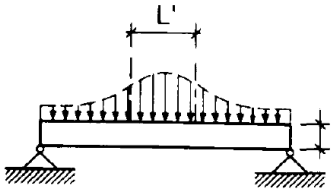
St Strouhal number as defined in E.1.3.2.

E.1.5.2.4 Effective correlation length factor K_w

(1) The effective correlation length factor, K_W, is given in Expression (E.8).

$$K_W = \frac{\sum_{j=1}^{n} \int_{L_j} |\Phi_{i,y}(s)|\,ds}{\sum_{j=1}^{m} \int_{\ell_j} |\Phi_{i,y}(s)|\,ds} \leq 0,6 \qquad \text{(E.8)}$$





where:

$\Phi_{i,y}$ is the mode shape i (see F.3).

L_j is the correlation length

ℓ_j is the length of the structure between two nodes (see Figure E.3); for cantilevered structures it is equal to the height of the structure

n is the number of regions where vortex excitation occurs at the same time (see Figure E.3)

m is the number of antinodes of the vibrating structure in the considered mode shape $\Phi_{i,y}$

s is the coordinate defined in Table E.5.

E.1.5.2.3 Correlation length L_j

(1) The correlation length L_j, should be positioned in the range of antinodes. Examples are given in Figure E.3. For guyed masts and continuous multispan bridges special advice is necessary.

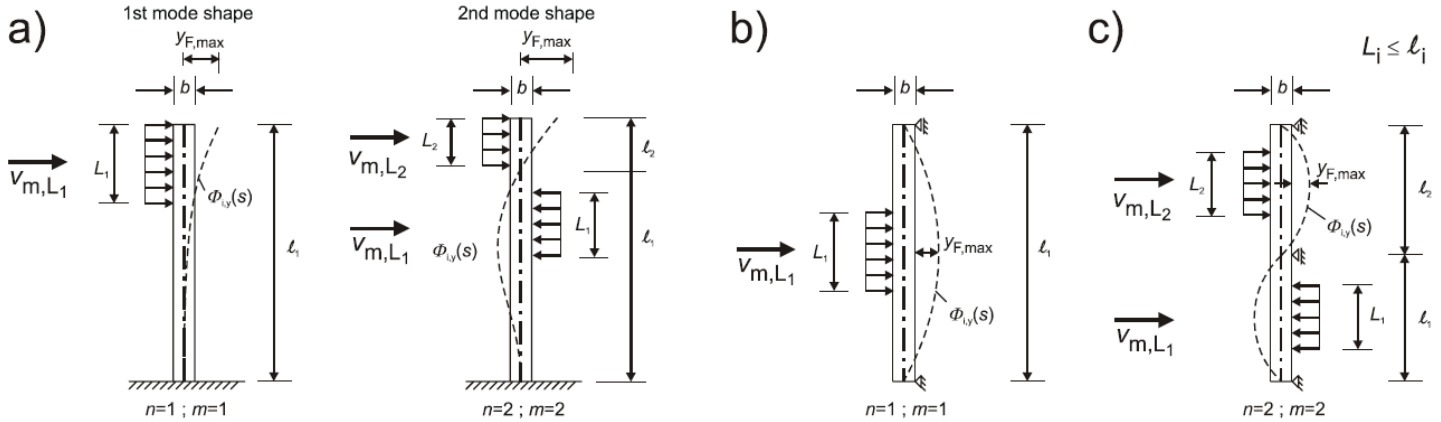


Table E.4 — Effective correlation length L_j as a function of vibration amplitude $y_F(s_j)$

$y_F(s_j)/b$	L_j / b
< 0,1	6
0,1 to 0,6	$4{,}8+12 \cdot \frac{y_F(s_j)}{b}$
> 0,6	12

E.1.5.2.5 Mode shape factor K

(1) The mode shape factor K is given in Expression (E.9).

$$K = \frac{\sum_{j=1}^{m} \int_{\ell_j} |\, \Phi_{i,y}(s) \,|\, ds}{4 \cdot \pi \cdot \sum_{j=1}^{m} \int_{\ell_j} \Phi_{i,y}^2(s)\, ds} \qquad \text{(E.9)}$$

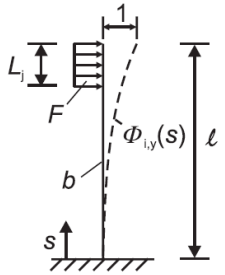
where:

m is defined in E.1.5.2.4 (1)

$\Phi_{i,y}(s)$ is the cross-wind mode shape i (see F.3)

ℓ_j is the length of the structure between two nodes (see Figure E.3)

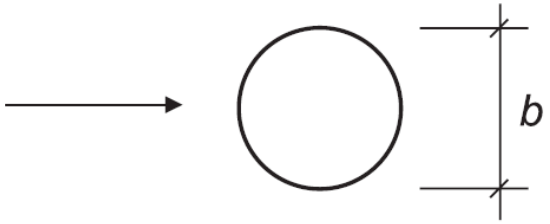
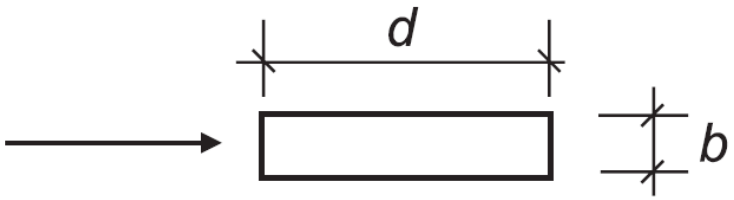
Table E.5 — Correlation length factor K_W and mode shape factor K for some simple structures

Structure	mode shape $\Phi_{i,y}(s)$	K_W	K
	see F.3 with ζ = 2,0 n = 1 ;m = 1	$3 \cdot \frac{L_j / b}{\lambda} \cdot \left[1 - \frac{L_j / b}{\lambda} + \frac{1}{3} \cdot \left(\frac{L_j / b}{\lambda} \right)^2 \right]$	0,13

E.1.5.2.2 Lateral force coefficient c_{lat}

(1) The basic value, $c_{lat,0}$, of the lateral force coefficient is given in Table E.2.

Table E.2 — Basic value of the lateral force coefficient $c_{lat,0}$ for different cross- sections

Cross-section	$c_{lat,0}$
 for all *Re*-numbers	from Figure E.2
 $0,5 \leq d/b \leq 10$	1,1

E.1.5.2.2 Lateral force coefficient c_{lat}

Table E.3 — Lateral force coefficient c_{lat} versus critical wind velocity ratio, $v_{crit,i}/v_{m,Lj}$:

Critical wind velocity ratio	c_{lat}
$\frac{v_{crit,i}}{v_{m,Lj}} \leq 0{,}83$	$c_{lat} = c_{lat,0}$
$0{,}83 \leq \frac{v_{crit,i}}{v_{m,Lj}} < 1{,}25$	$c_{lat} = \left(3 - 2{,}4 \cdot \frac{v_{crit,i}}{v_{m,Lj}}\right) \cdot c_{lat,0}$
$1{,}25 \leq \frac{v_{crit,i}}{v_{m,Lj}}$	$c_{lat} = 0$

where:

$c_{lat,0}$ is the basic value of c_{lat} as given in Table E.2 and, for circular cylinders, in Figure E.2

$v_{crit,i}$ is the critical wind velocity (see Expression (E.1))

$v_{m,Lj}$ is the mean wind velocity (see 4.2) in the centre of the effective correlation length as defined in Figure E.3

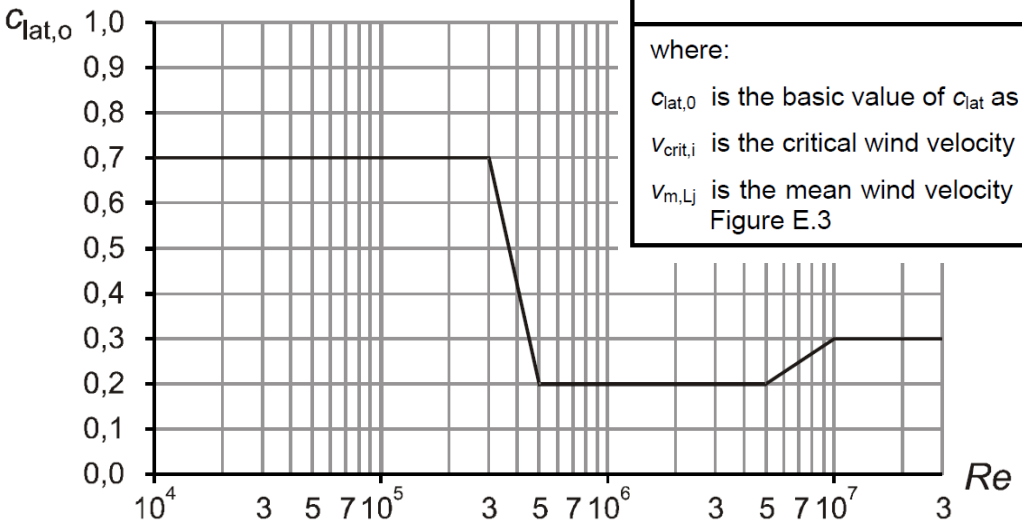
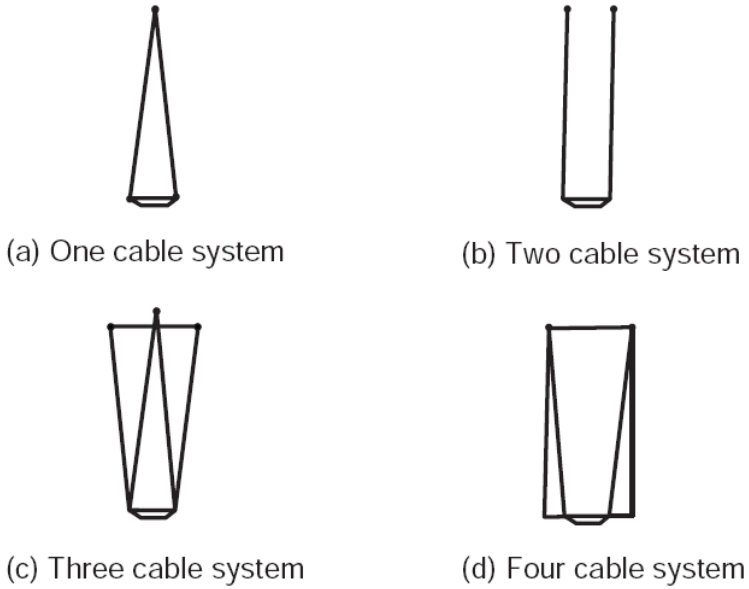


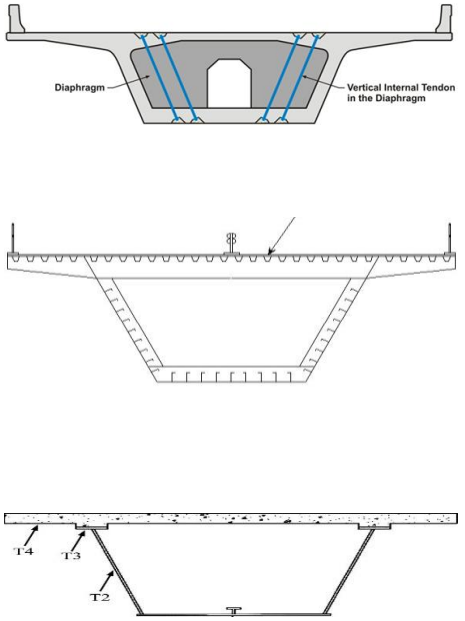
Figure E.2 — Basic value of the lateral force coefficient $c_{lat,0}$ versus Reynolds number $Re(v_{crit,i})$ for circular cylinders, see E.1.3.4

Wind-induced Vibration Control

1. structural modification (stiffness, damping and mass);
2. aerodynamic measures (guide vanes, shape change); and
3. mechanical measures (passive, active control system).



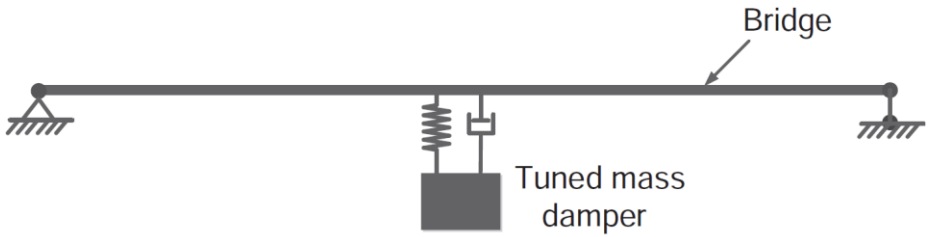
Cable system configuration

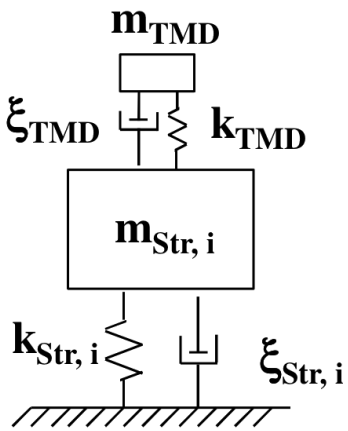


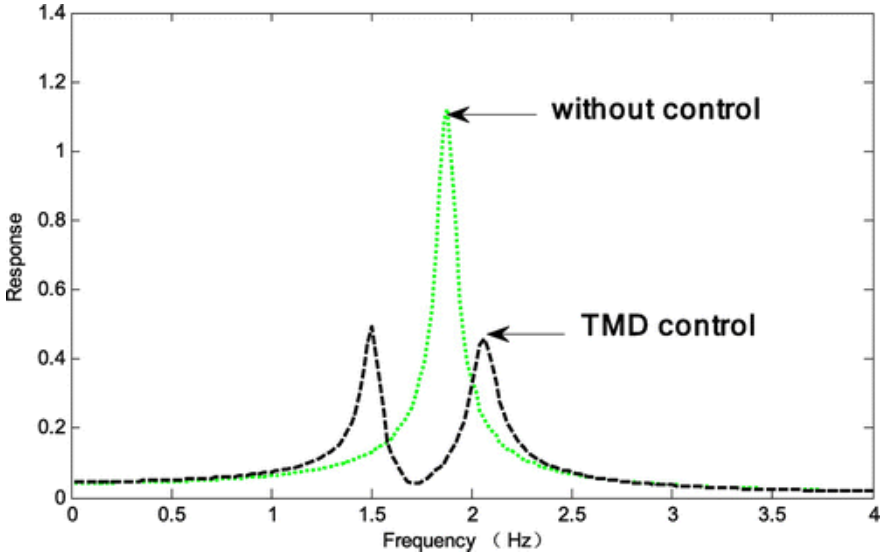
Main span material

1. Structural Modifications

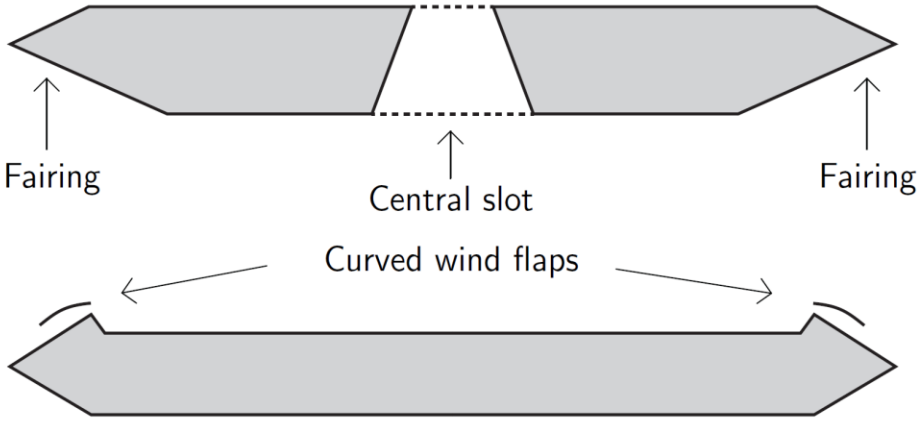
Tuned Mass Dampers (TMD)

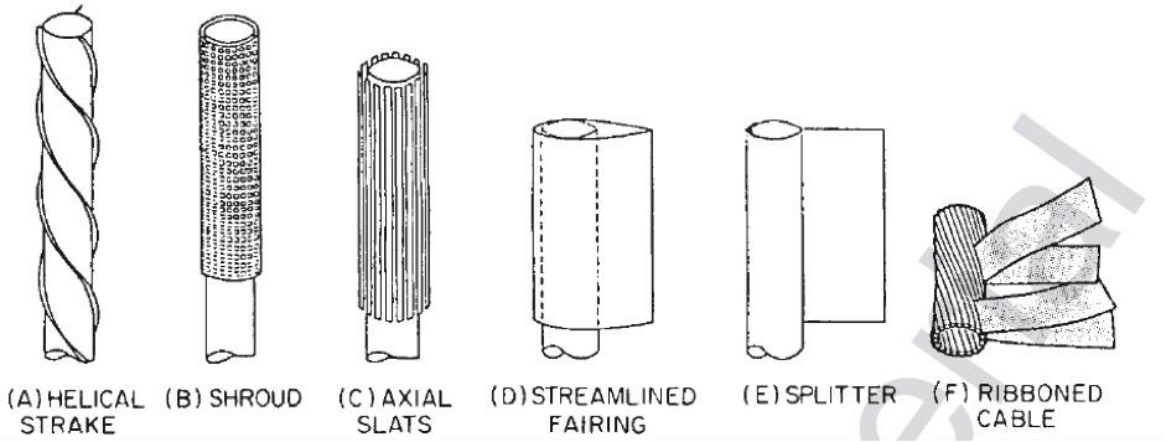


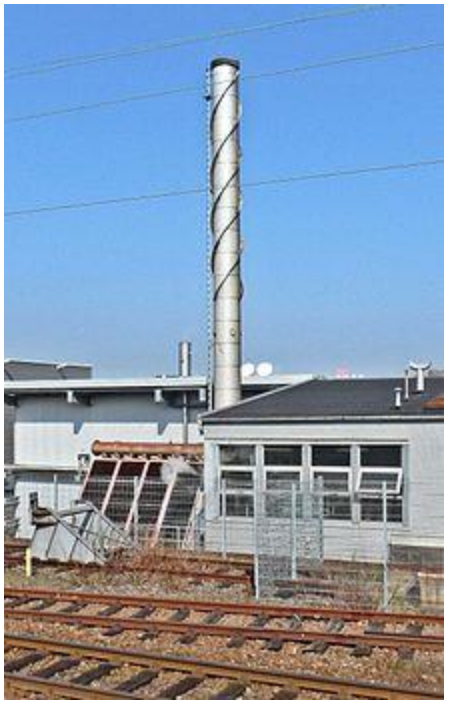




2. Aerodynamic Measures

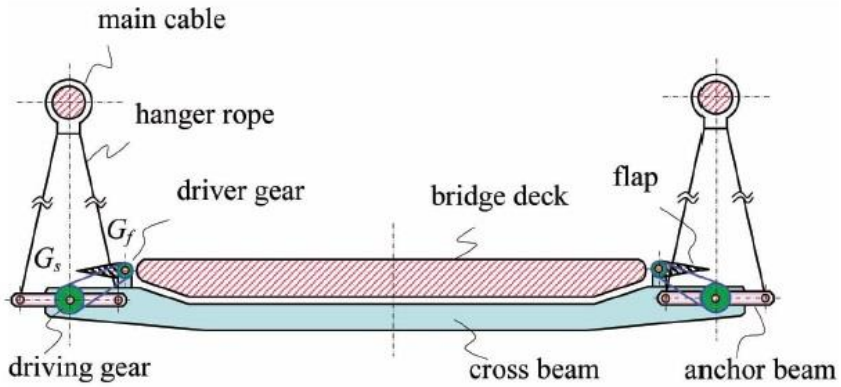




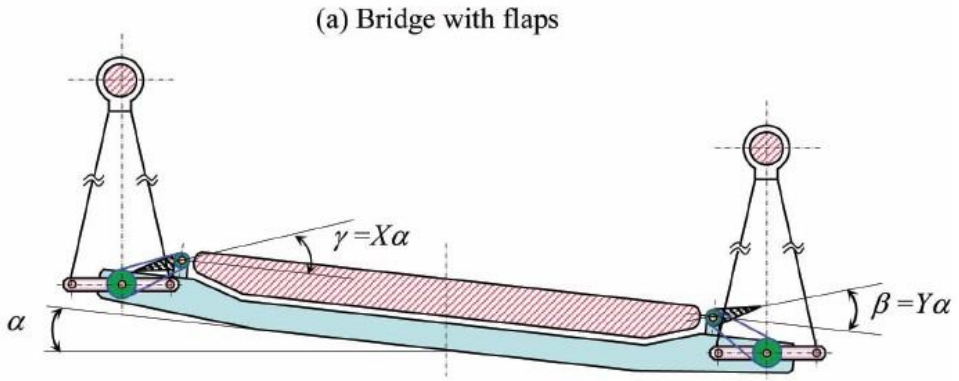


3. Mechanical Measures

- Passive control flaps

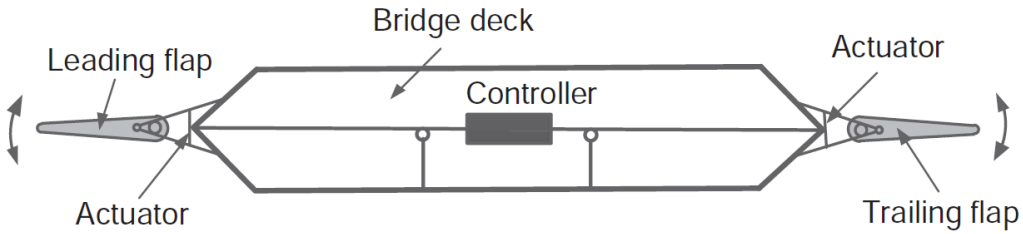


(a) Bridge with flaps



(b) Deformation of bridge and flaps

- Active control flaps

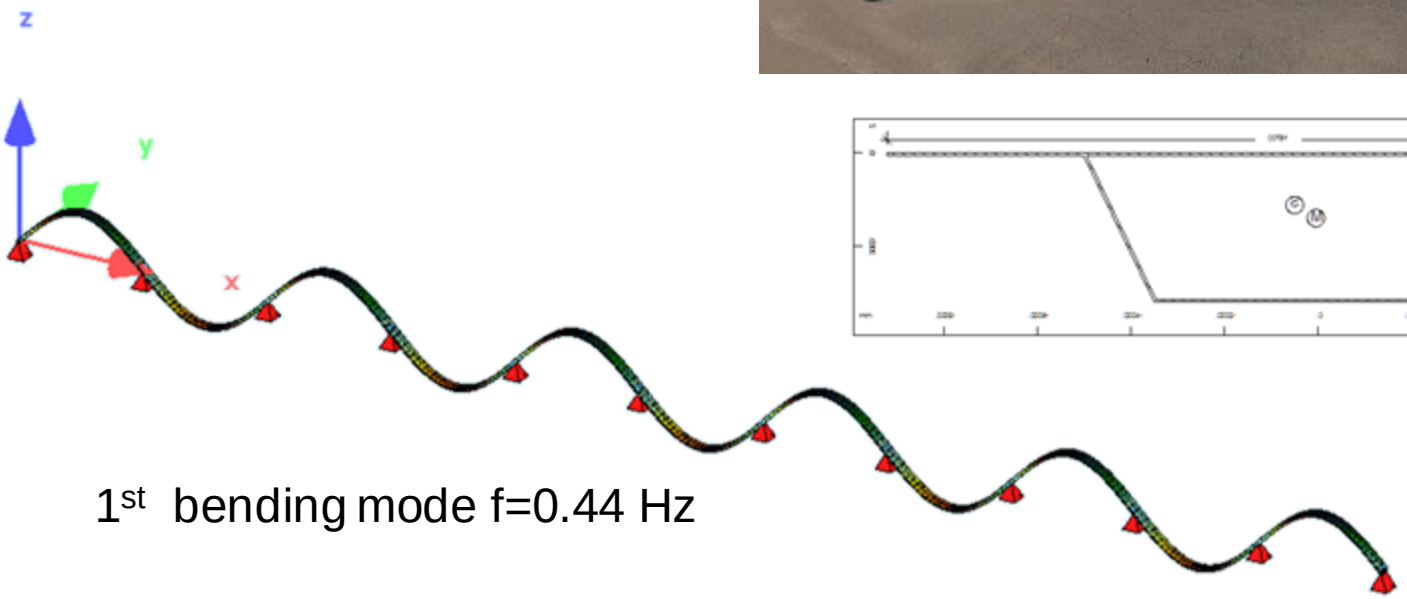


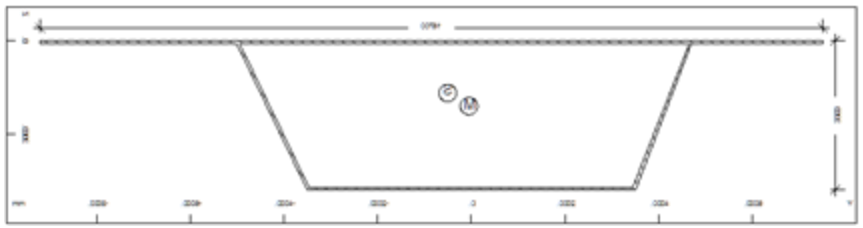
Volgograd Bridge, Russia
(Steel deck box girder, 2009)

Volgograd Bridge, Russia

Finite Element Model

Span length : 155 m
number of spans : 11
Deck section : Girder box

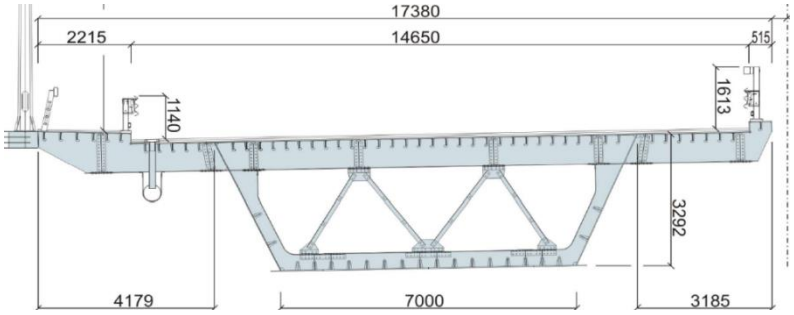


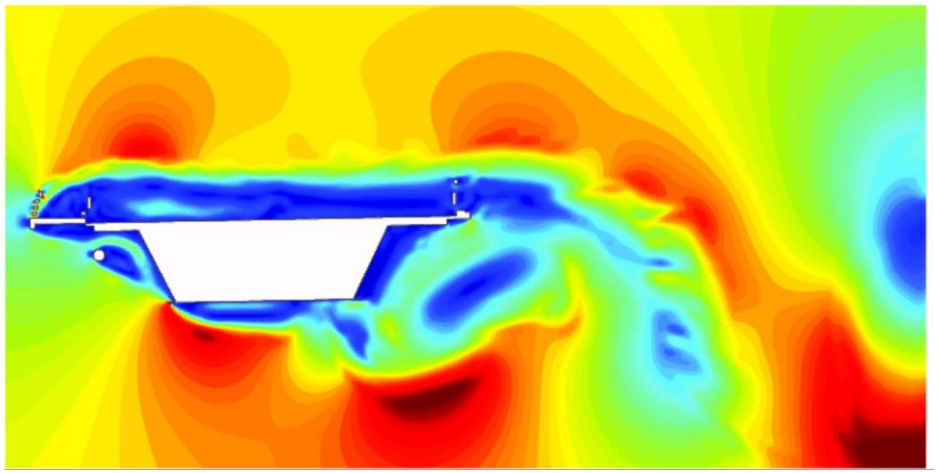


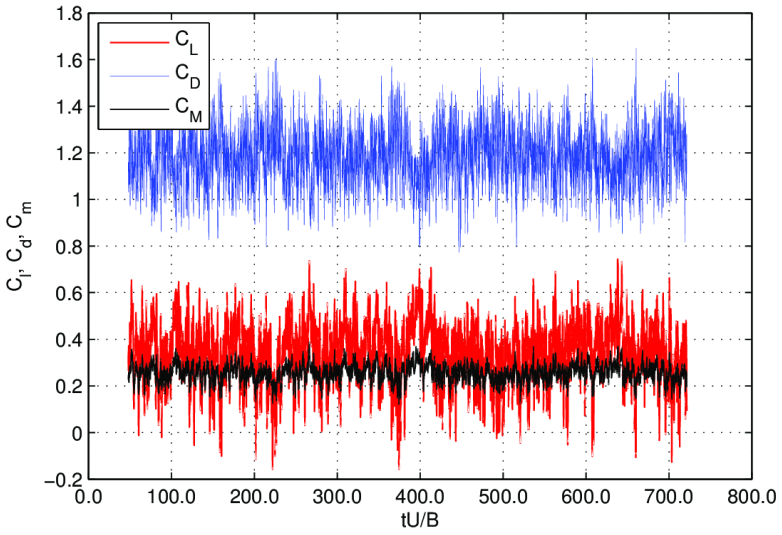
1st bending mode f=0.44 Hz

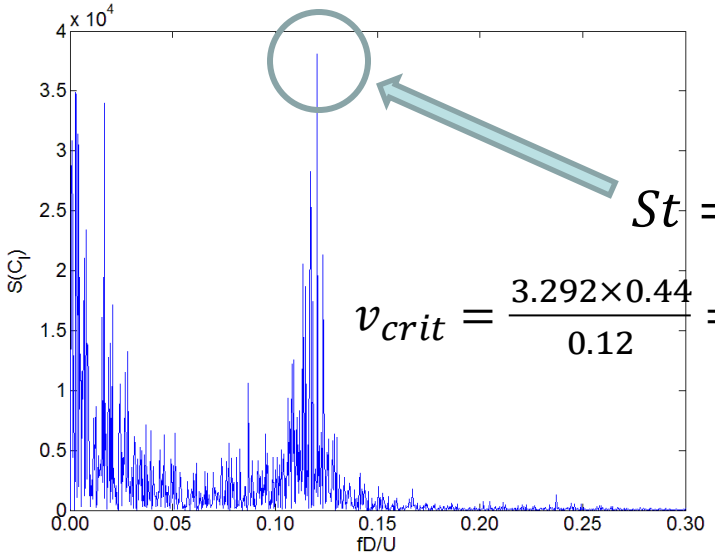
CFD simulations

- Static analysis







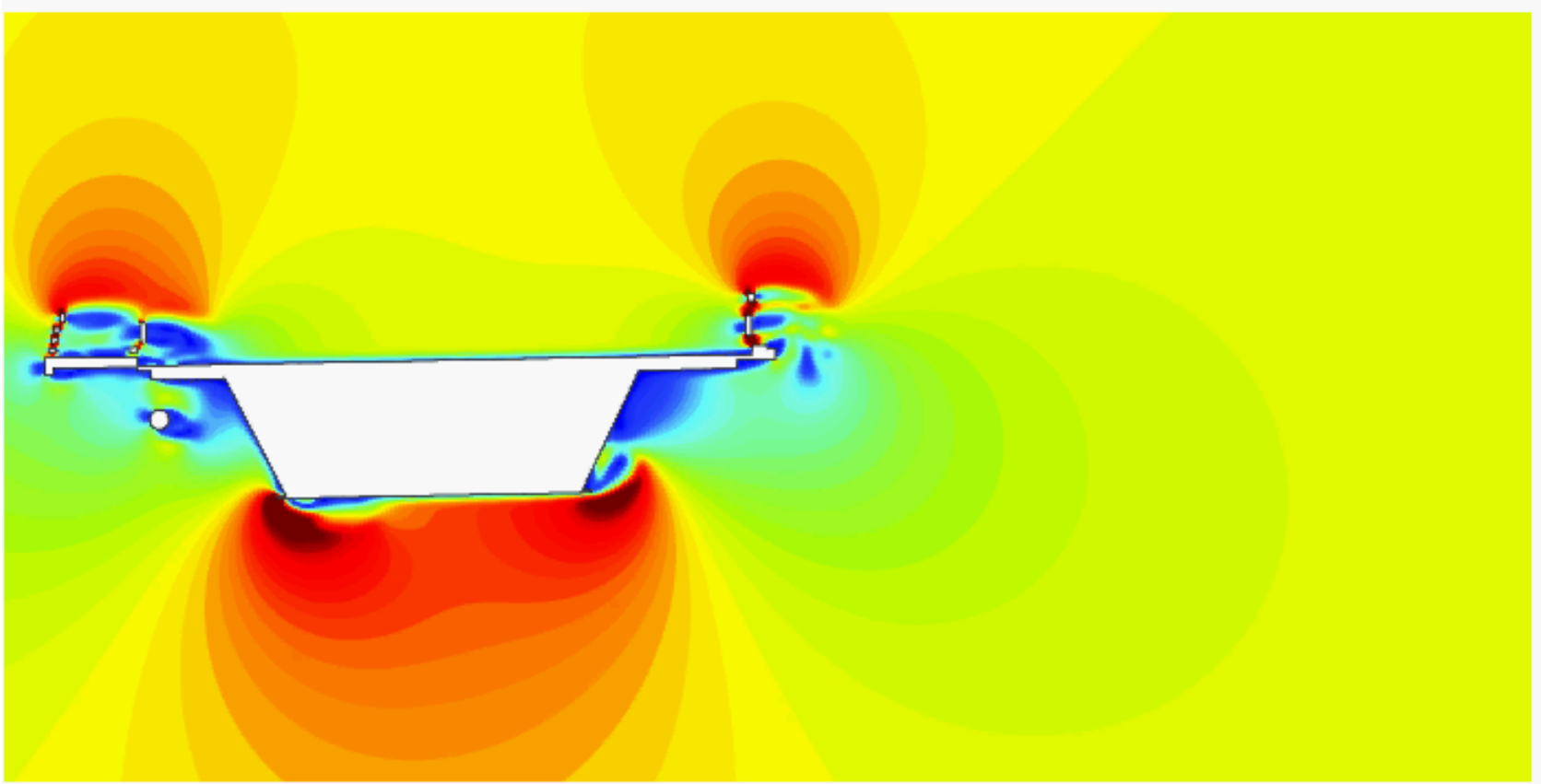


$$St = 0.12$$

$$v_{crit} = \frac{3.292 \times 0.44}{0.12} = 12.1 \text{ m/s}$$

CFD simulations

- Dynamic analysis



Tuned Mass Dampers (TMD)

Adaptive TMD: fitted with an adaptive damping element automatically controlled in real time

Tuned Mass Dampers (TMD)

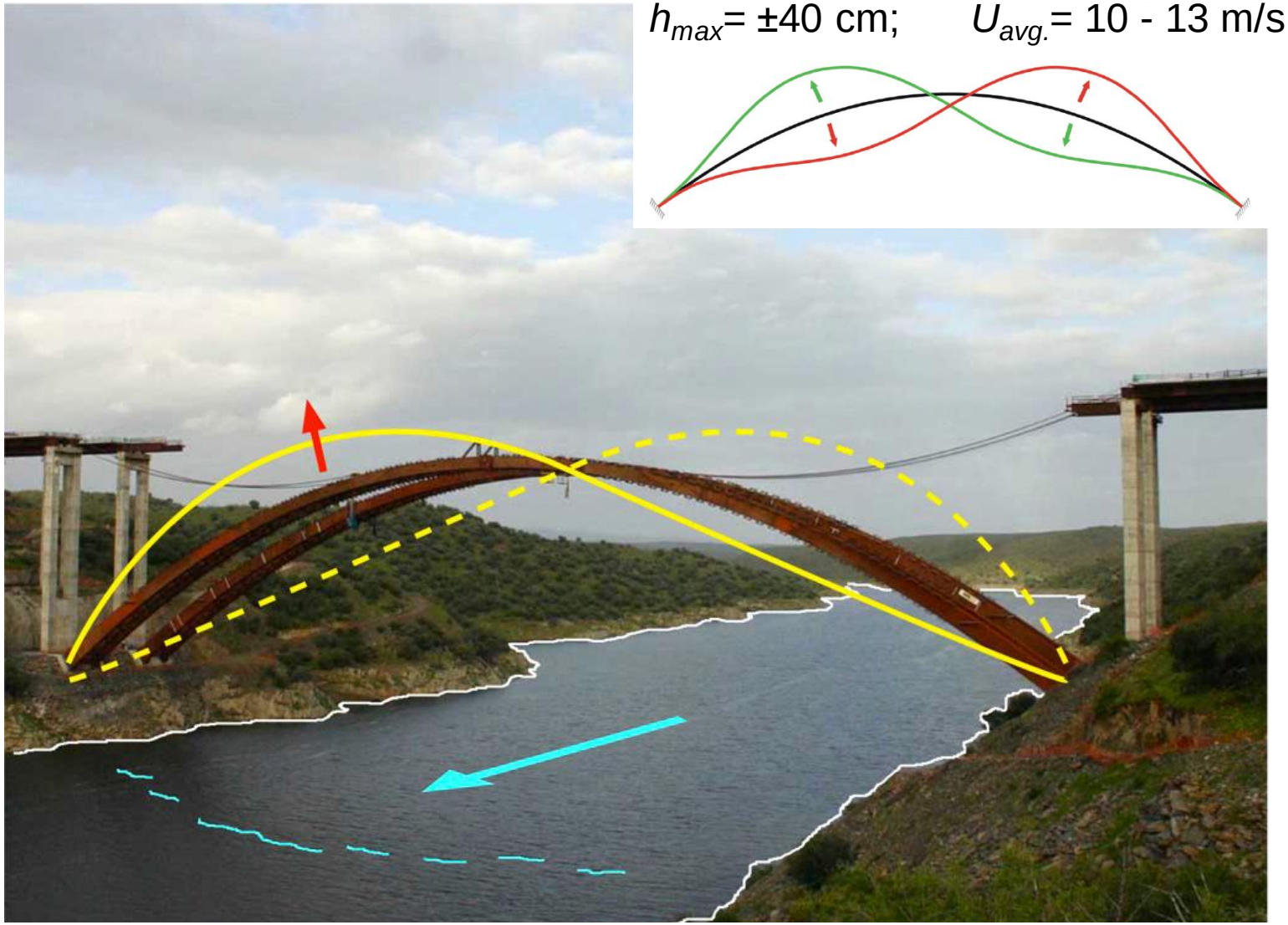
Alconétar Bridge, Spain
(220m arch span, 2006)

Construction stage

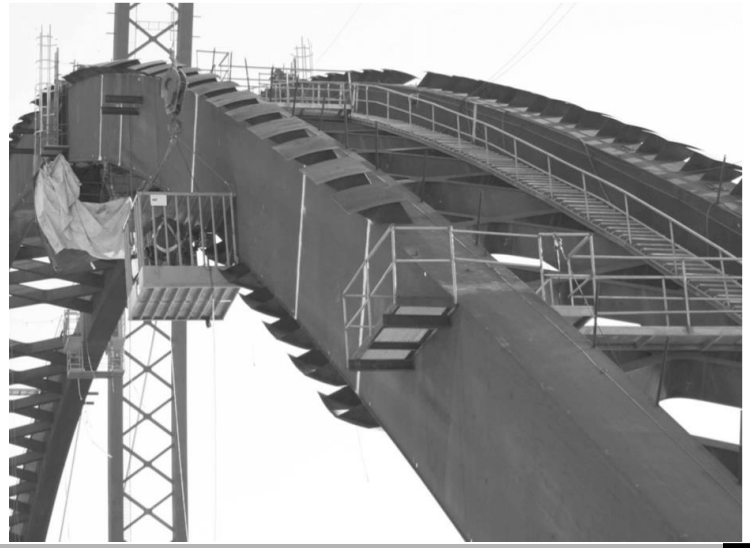
VIV at Construction Stage

VIV at Construction Stage

h_{max}= ±40 cm; $U_{avg.}$= 10 - 13 m/s



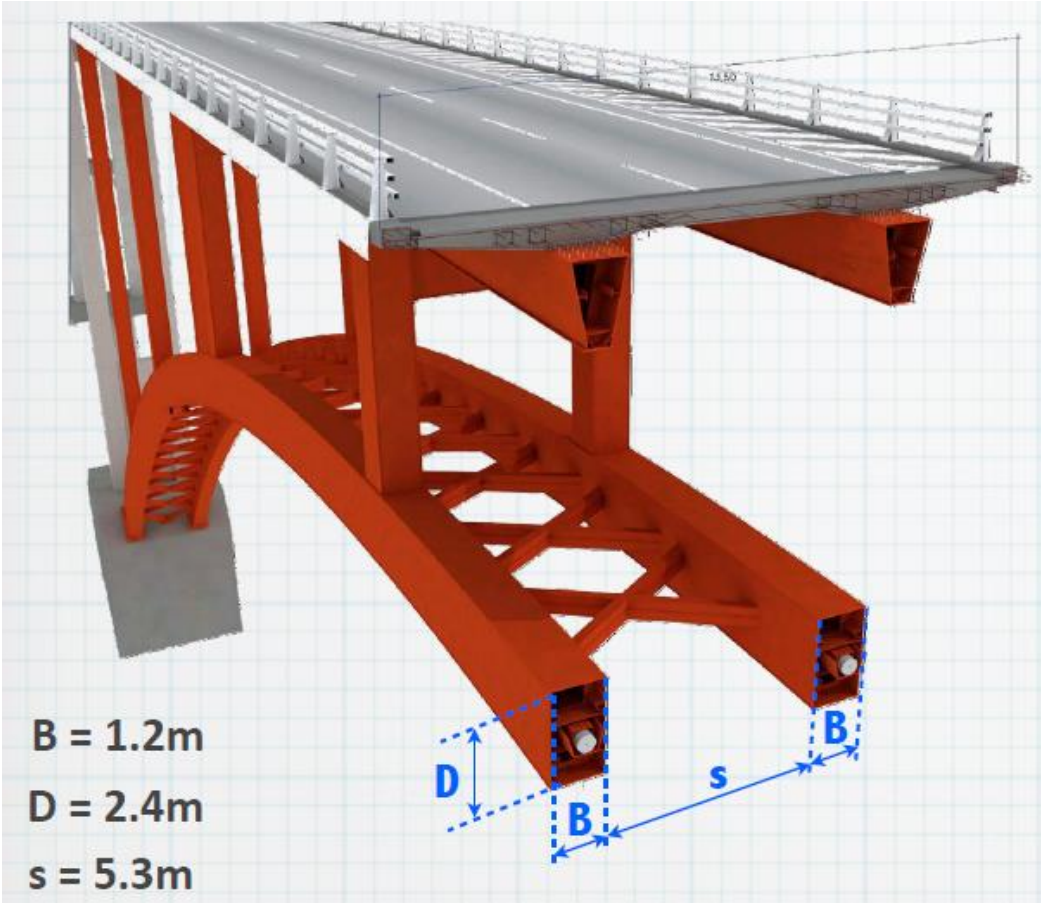
Control Measures



Alconétar Bridge

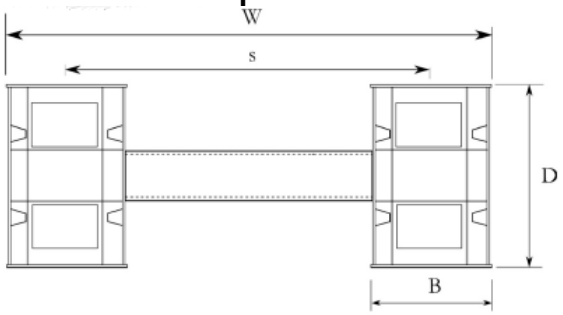
Dynamic properties (1DOF):

Mass, $m = 1130$ kg/m
Frequency, $f_n = 0.7$ Hz
Damping ratio, $\xi = 0.3$ %

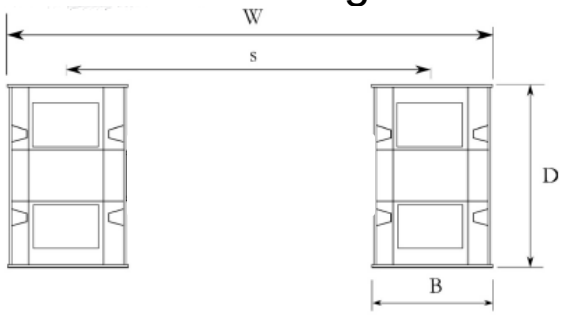


CFD simulations

H-shape



Twin rectangles

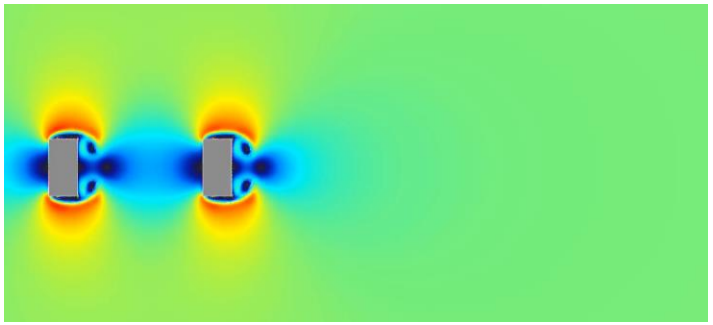


- Strouhal number

$St = 0.205$ $St = 0.125$

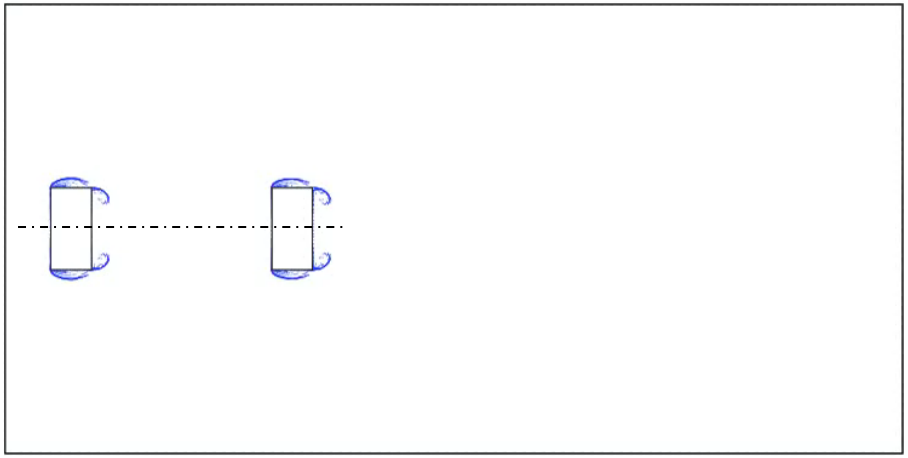
- Critical wind velocity, $v_{crit} = \frac{D \times f_n}{St}$

$v_{crit} = \frac{2.4 \times 0.69}{0.205} = 8.0$ m/s $v_{crit} = \frac{2.4 \times 0.69}{0.125} = 13.4$ m/s

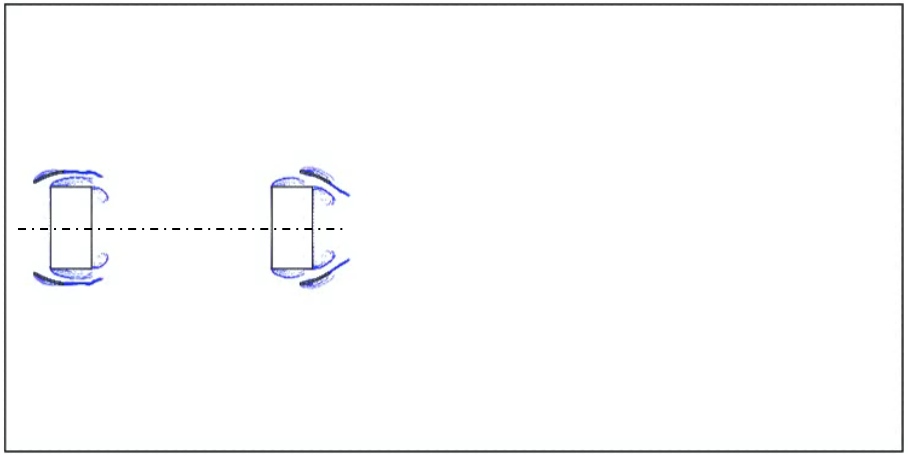


CFD simulations

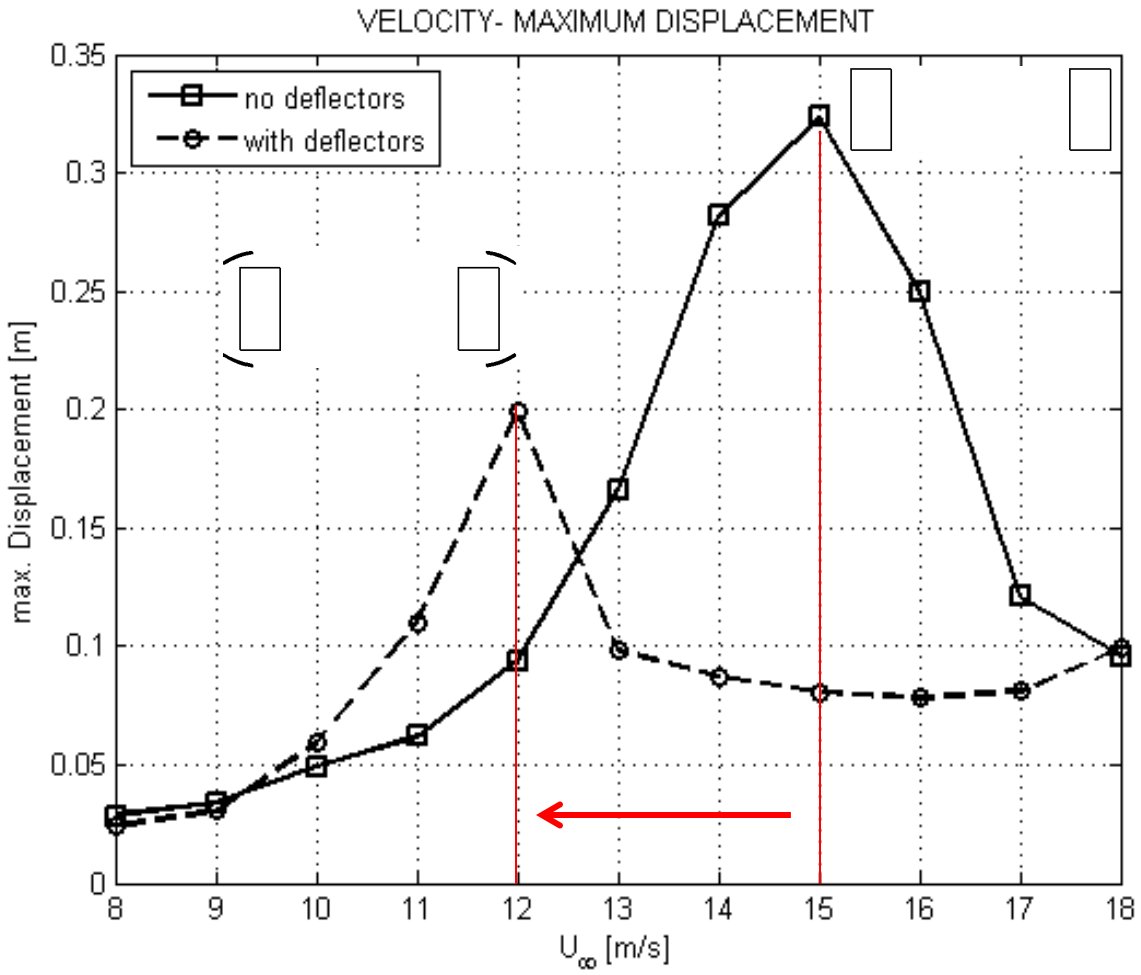
- without deflectors, U=15 m/s



- with deflectors, U=12 m/s



Peak Displacements



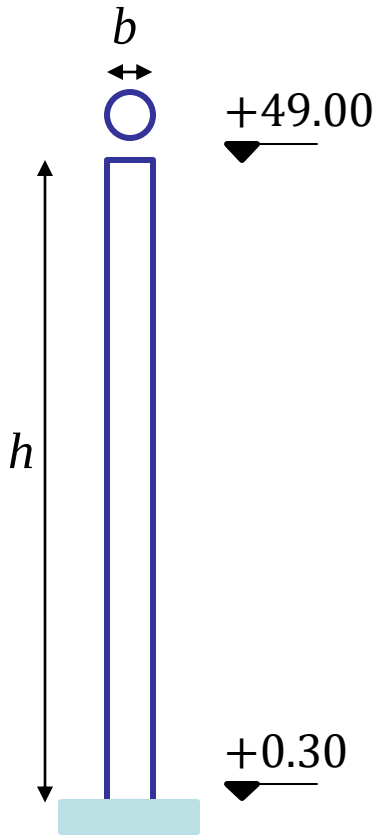
Example: Chimney

Given:

Section – circular
Material – steel stack with two liners
with external thermal insulation

Diameter, $b = 1620$ mm
Height, $h = 48.70$ m

Structural mass, $W_s = 239$ kg/m
Additional mass, $\Delta W = 121$ kg/m



Required:

cross wind amplitudes – y_{max}
Based on Eurocode

Procedure:

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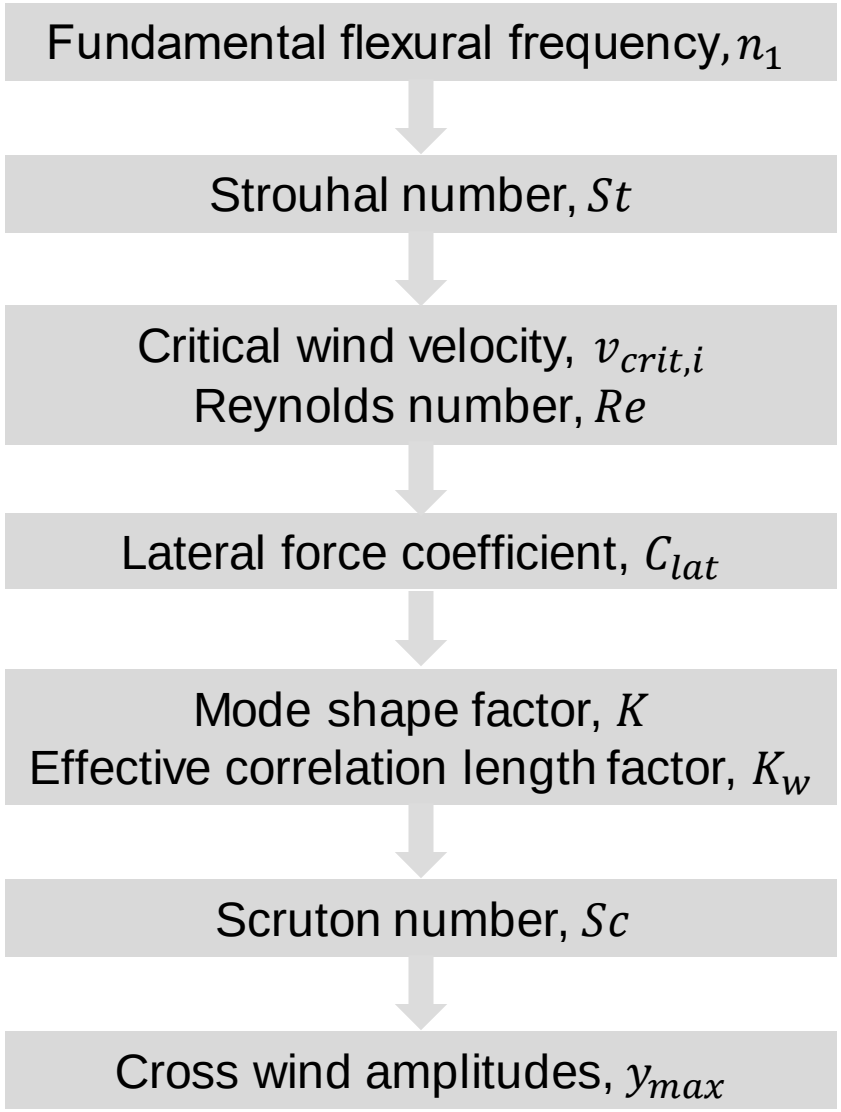


Example: Chimney

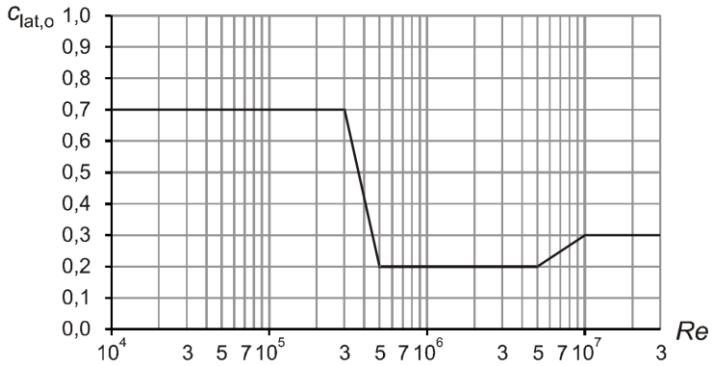
Solution:

Effective height of the chimney [Eq. (F.4)]
$h_{eff} = h = 48.70$ m

Total weight of the chimney [Sec. F.2, (3)]
$W_t = W_s + \Delta W = 239 + 121 = 360$ kg/m

Factor ε_1 [Sec. F.2, (3)]
$\varepsilon_1 = 1000$
Fundamental flexural frequency [Eq. (F.3)]
$n_1 = \frac{\varepsilon_1 \times b}{h_{eff}^2} \sqrt{\frac{W_s}{W_t}} = \frac{1000 \times 1.62}{48.70^2} \sqrt{\frac{239}{360}} = 0.838$ Hz

Strouhal number [Table E.1]
$St = 0.2$
Critical wind velocity for bending vibration mode i [Eq. (E.2)]
$v_{crit,i} = \frac{b \times n_{i,y}}{St} = \frac{1.62 \times 0.69}{0.2} = 5.6$ m/s



Example: Chimney

Kinematic viscosity of the air [Sec. E.1.3.4 (1)]
$\nu = 15 \times 10^{-6}$m^2/s

Reynolds number at the critical wind velocity [Eq. (E.5)]
$Re(v_{crit,i}) = \frac{b \times v_{crit,i}}{\nu} = \frac{1.62 \times 5.6}{15 \times 10^{-6}} = 6.0 \times 10^5$

Basic value, clat,0, of the lateral force coefficient [Fig. E.2]
$C_{lat,0}(Re) = 0.2$
Lateral force coefficient [Sec. E.1.5.2.2]
$C_{lat} = C_{lat,0} = 0.2$

Mode shape factor [Sec. E.1.5.2.5]
$K = 0.13$

Effective correlation length factor [Sec. E.1.5.2.4, Table E.5]
$\lambda = \frac{l}{b} = \frac{48.70}{1.62} = 30.1$
$K_w = \left(1 - \frac{6}{\lambda} + \frac{12}{\lambda^2}\right).\frac{18}{\lambda} = \left(1 - \frac{6}{30.1} + \frac{12}{30.1^2}\right).\frac{18}{30.30.1} = 0.487$

Example: Chimney

equivalent mass *m*e per unit length for mode i [Sec. F.4 (1)]
$m_{i,e} = 360$ kg/m

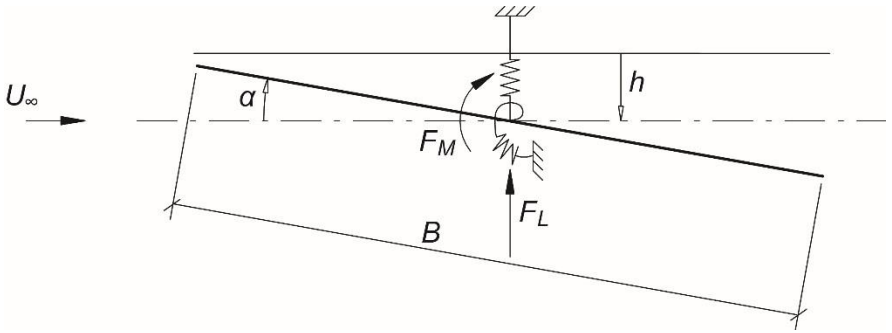
logarithmic decrement of structural damping [Sec. F.5, Table F.2]
$\delta_s = 0.025$ with $\frac{h}{b} = 30.1 > 26$

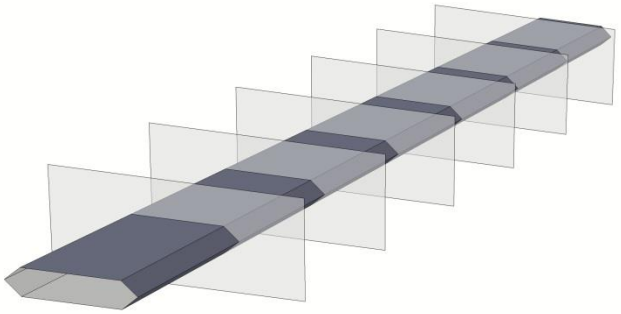
Scruton number [Sec. E.1.3.3]
$Sc = \frac{2\delta_s m_{i,e}}{\rho b^2} = \frac{2 \times 0.025 \times 360}{1.25 \times 1.62^2} = 5.487$

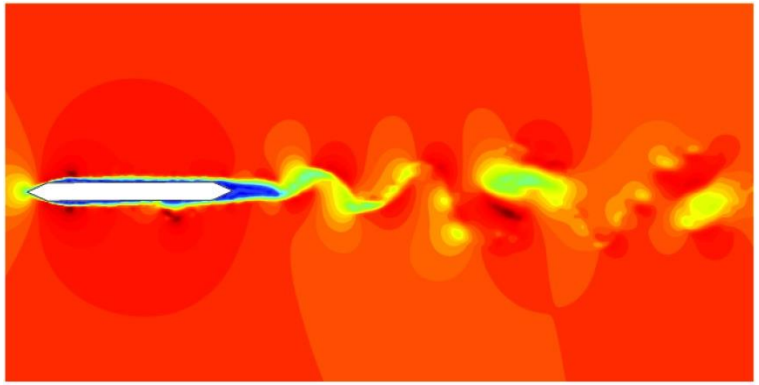
cross wind amplitudes [Sec. E.1.5.2.1]

$$\frac{y_{max}}{b} = \frac{1}{St^2} . \frac{1}{Sc} . K . K_w . C_{lat} = \frac{1}{0.2^2} \times \frac{1}{5.487} \times 0.13 \times 0.487 \times 0.2 = 0.058$$

or
$y_{max} = b \times 0.058 = 1.62 \times 0.058 = 0.094$ m or **94 mm**







Thank you